Pricing of Options on Defty

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INTRODUCTION

A foreign portfolio investor needs to evaluate investment in an equity market from a different perspective as compared to a local investor. This proposal first presents an investment attractiveness model for foreign investment in emerging markets. This model uses the Treynor ratio as a benchmark and decomposes its numerator to identify three factors driving net returns - Excess return in Indian equity market over the Indian risk-free rate, Interest rate differential between India and the investor's home country, say US, and Exchange rate returns in the following manner:

$$Net\ Return = \left(R_{fI} - R_{fUS}\right) + \left(R_s - R_{fI}\right) + \tilde{s}(1 + R_s)$$

Analyzing the returns from Indian equity in this framework (Exhibit 2 in Appendix 1), one observes that though the excess rupee returns from Nifty have remained predominantly stable around an annual 25% level, the net return has varied between 26% and close to 40%, primarily on account of exchange rate fluctuations.

This analysis proves that the Nifty index is not a robust measure of portfolio performance for a foreign investor in the Indian equity market since it does not capture the effect of exchange rate. This was, possibly, the rationale behind formulation of the Defty index. Moreover, a foreign investor currently needs to hedge itself both in the equity market and in the FX market, which might be cumbersome and involve higher transaction costs. The introduction of derivative instruments like options on Defty by NSE could provide the foreign investor with a direct instrument to hedge its positions in the Indian market. As a first attempt to price a cross-currency derivative with Indian equity as underlying, this research study aims to develop pricing models for options on Defty.

RESEARCH OBJECTIVE

A foreign investor's portfolio returns in our market are affected both by returns on the market as well as the USD/INR exchange rate. The net return to a US investor from Indian markets has fluctuated primarily due to exchange rate movements in the past three years. While Nifty is considered as an indicator of Indian equity market performance, the Defty or dollar denominated Nifty index would prove to be a more useful measure for any foreign investor for monitoring Indian investment performance. Though NSE lists the Defty index, no derivative products have been developed on it as yet. We contend that the introduction of derivatives on the Defty index would not only provide a useful risk hedging instrument for foreign investors but as a consequence also increase the investment attractiveness of Indian equity markets. A key research issue associated with this index is regarding the pricing of options on it. We address this problem and suggest and implement a methodology for pricing European options on Defty.

LITERATURE SURVEY

The pricing of cross currency and multi-asset options has attracted significant research interest in the past few years. Numerous papers like that by (Benninga, Bjork, & Wiener, 2002)have evaluated prices of options on foreign assets denominated in domestic currency through the standard Black Scholes approach. This approach has been driven by assuming the domestic currency denominated asset price to follow a lognormal distribution. As with other options, this approach assumes a constant volatility assumption on the price process.

However stock prices have been well documented to exhibit time varying variances, skewness and leptokurtosis. Time series data is often characterised by volatility clustering i.e. periods of excess volatility followed by periods of relative calm. A historical volatility measure is unable to identify these effects. A study by (Harikumar, Boyrie, & Pak) and examines the daily exchange rates and finds an overwhelming presence of volatility clustering. The ARCH / GARCH models and their variants have been proposed to model this time varying volatility behaviour by a number of researchers like (Engle, 1982), (Bollerslev, Engle, & Nelson, 1994), (Bollerslev, 1987), (Nelson, 1989). Various other researchers have analyzed the performance of these models on different asset price data. It has generally been found that the GARCH(1,1) model with the threshold leverage effect is adequate for characterizing most financial time series. The GARCH frameworks have also been applied for pricing of options by (Engle & Chowdhury, 1992) and various other researchers.

(Duan, 1995) developed a GARCH based option pricing model that could be applied to the pricing of cross-currency options by incorporating a risk premium parameter. The model uses a change of numeraire based approach to develop a set of locally risk neutral valuation relationships (LRNVR). This model was also found to show significantly better performance than the Black Scholes approach for pricing of short term options by (Chaudhury, Jason, & Z, 1996) and in option pricing by (Myers & Hanson, 1993). Moreover, (Bakshi, Cao, & Chen, 1997) have found that overall, incorporating stochastic volatility and jumps is important for pricing and internal consistency and that often, it is stochastic volatility modelling that yields the best results. In cases where the volatility process is bivariate, (Ritchken & Trevor, 1999) suggest that a bi-variate GARCH model, with correlation between the underlying, explains the process quite sufficiently.

PROPOSED PRICING MODELS

The Defty index is defined as follows:

$$Defty = \frac{Nifty_t}{S_t} \times S_0$$

where S_t is the current USD/INR exchange rate.

The base date is taken to be 3rd November, 1995 when the Nifty value was 1000. S0 for this date was 34.65. More conveniently, Defty can also be expressed as follows:

$$Defty = D_t = N_t E_t S_0$$

where N_t is the value of the Nifty index and E_t $(= 1/S_t)$ is the INR/USD exchange rate.

Thus the Defty value involves two price values – Nifty and the exchange rate. Since a stock index value can be considered as an asset in itself6, it can be assumed to follow a Geometric Brownian motion price process. Similarly exchange rate is also assumed to follow a log-normal distribution:

$$\frac{dN}{N} = \mu_N dt + \sigma_N dz_N$$

$$\frac{dE}{E} = \mu_E dt + \sigma_E dz_E$$

It can be shown that the Defty value, D would also follow a Geometric Brownian motion price process as follows:

$$\frac{dD}{D} = (\mu_N + \mu_E + \rho\sigma_N\sigma_E)dt + (\sigma_N^2 + \sigma_E^2 + 2\rho\sigma_N\sigma_E)dz$$

where ρ is the correlation between exchange rate and Nifty price processes. Thus,

$$\mu_D = \mu_N + \mu_E + \rho\sigma_N\sigma_E$$

$$\sigma_D^2 = \sigma_N^2 + \sigma_E^2 + 2\rho\sigma_N\sigma_E$$

This suggests the applicability of the Black Scholes model to Defty option pricing. Note that the risk neutral option price here would correspond to the US risk-neutral measure since the option would have a USD denominated pay-off. Thus a call option on the Defty index with a continuous dividend yield, q (same as the dividend yield of Nifty on the corresponding period) would be valued as follows:

$$c = De^{-qt}N(d_1) - Xe^{-rt}N(d_2)$$

Where

$$d_1 = \frac{Ln\frac{D}{X} + (r - q + \sigma_D^2)T}{\sigma_D\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma_D \sqrt{T}$$

D and X are the spot Defty value and strike Defty value respectively. The volatility of the Defty index would be valued through both: simplistic historical volatility estimation and through a forecasted estimate of the GARCH model over the duration of the option. Various standard GARCH based models such as EGARCH, PARCH and TGARCH would be employed and tested for forecasting power on a hold out sample of Defty prices. The model showing the best forecast would be used for the valuation. The call price *c* is denominated in Defty index points and can be converted into USD depending on the option contract specifications. Though the estimation of volatility using GARCH in the Black Scholes equation uses a forecasted value of volatility, it still does not account for heteroskedaticity in exchange rate and Nifty returns. A direct simulation based GARCH pricing on Defty data is also ruled out since Nifty returns and exchange rate data are expected to show different behaviour. Thus a bi-variate GARCH model (Chen, Duan, & Hung, 1999) and (Duan & Wei, 1999) needs to be used to characterize both exchange rate and Nifty series:

$$Ln\frac{N_t}{N_{t-1}} = \delta_t^N - \frac{1}{2}h_t + u_t$$

$$h_t = \beta_0 + \beta_1(u_{t-1} - a)^2 + \beta_2 h_{t-1}$$

$$Ln\frac{E_t}{E_{t-1}} = \delta_t^E - \frac{1}{2}g_t + v_t$$

$$g_t = \beta_0 + \beta_1(v_{t-1} - b)^2 + \beta_2 g_{t-1}$$

where *h* and *g* are the conditional variances of Nifty and exchange rate series respectively, *a* and *b* are the asymmetric leverage factors.

Note that δ_t^N and δ_t^E are the risk neutral drift rates of Nifty (INR denominated) and exchange rate returns respectively under the US money market risk neutral measure. Under Ito calculus considerations it can be shown that:

$$\delta_t^E = r_{US} - r_{IN}$$

$$\delta_t^N = r_{IN} - q - \rho\sqrt{h_t g_t}$$

where r_{US} and r_{IN} are the risk free rates in the US and India respectively.

The δ_t^N term above includes the effect of correlation between exchange rate and Nifty series and thus the two pair equations need to be estimated together. This model can be estimated in the following form (Tsay, 2002):

$$\begin{bmatrix}\sigma_{11,t}\\ \sigma_{22,t}\end{bmatrix} = \begin{bmatrix}\alpha_{10}\\ \alpha_{20}\end{bmatrix} + \begin{bmatrix}\alpha_{11} & \alpha_{12}\\ \alpha_{21} & \alpha_{22}\end{bmatrix}\begin{bmatrix}a_{1,t-1}^2\\ a_{2,t-1}^2\end{bmatrix} + \begin{bmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{bmatrix}\begin{bmatrix}\sigma_{11,t-1}\\ \sigma_{22,t-1}\end{bmatrix}$$

Where

$$a_t^2 = \alpha_0 + (\alpha_1 + \beta_1)a_{t-1}^2 + \eta_t - \beta_1\eta_{t-1}$$

The GARCH model would be estimated using a Maximum Likelihood Estimation (MLE) of conditional returns using the robust Berndt-Hall-Hall-Hausmann (BHHH) algorithm. The estimated model would be simulated using Monte Carlo simulations over the option duration to yield expected values of N_T and E_T.

Finally the call option price would be:

$$c = e^{-rt}\max\left(E[D_T] - X, 0\right)$$

The expected value of D_T can be calculated using the following co-variance relationship:

$$Cov[N_T, E_T] = E[N_T.E_T] - E[N_T].E[E_T]$$

Thus,

$$E[D_T] = S_0 E[N_T.E_T] = S_0\{Cov[N_T, E_T] + E[N_T].E[E_T]\}$$

The call prices would be calculated using the above two models for different durations and strike prices. More meaningful conclusions can be made from the comparisons if the models are tested against actual market price data of options on similar indices (e.g. dollar denominated Nikkei).

DATA ANALYSIS

Defty daily closing price data is used for pricing using the Black Scholes' based model. To effectively model the GARCH estimation, a large number of data points, starting from May 2002 are considered9. For pricing through the bi-variate GARCH model, daily Nifty closing prices are obtained from the NSE website. To correctly equivalently model the Defty prices, daily exchange rates at the time corresponding to Nifty closing need to be used. These are obtained as implied exchange rates from closing Defty and Nifty index prices:

$$E_t = \frac{Defty_t}{N_t S_0}$$

The risk-free India and US rates are taken to be the 10-year treasury yield rate and approximated as 7.0% and 4.5% respectively.

To test the log-normality assumption implicit in the Black Scholes based model, a Quantile-Quantile plot was generated for testing normality of log returns of Defty and Nifty indexes. Both the series showed normality with a good significance level. An Anderson- Darling test on these two series also showed similar results. Thus, Black Scholes model can be employed for option valuation under the US money market risk neutral measure.

The Defty returns data also displayed heteroskedaticity. This was tested by regressing the returns functions against a linear Auto-Regressive function and subjecting the residuals to an ARCH-LM test which displayed significant p-values. Thus, GARCH based models should be employed for option valuation.

Volatility Estimation

Daily Defty closing values, obtained from the NSE site, from January 02, 2002 to July 31, 2007 are used to build the model in EViews. Daily logarithmic returns are calculated on this data. Before applying the GARCH family of models, the amenability of Defty returns data for such a model is checked.

We first start with random walk hypothesis and regress the returns on a constant distribution. However, applying the serial correlation test on the residuals shows significant dependence on lag terms. Therefore, a returns process of Auto-Regressive Order 1 and Moving Average Order 1 [ARMA(1,1)] is assumed while still retaining the homoskedasticity assumption. A serial correlation test on the residuals of this model fails and significance values for various auto-regressive terms shows the presence of auto-regression. To test heteroskedaticity in the returns data, an ARCH LM test is applied on the residuals of the AR model. The significance level for various residuals proves the presence of heteroskedaticity in data and lends itself to the application of the GARCH models. The returns equation in the GARCH model is still taken to be ARMA(1,1).

The GARCH(1,1), TGARCH of threshold order 1, EGARCH, ETGARCH with threshold of order 1 and PARCH models are applied on the returns data. All of the predicted models show stability with the sun of the ARCH and the GARCH coefficients being less than one. The returns data from January 02, 2002 to July 31, 2006 is used to estimate each of these models. The accuracy in forecasting returns is measured on a 3 month (August 31 – October 31, 2007) hold-out sample of actual Defty returns. The Root Mean Square Error (RMSE) measure is used to compare the performance of these models. The GARCH(1,1) model exhibits the best performance with minimum RMSE of 0.009015. The results of the comparison have been included in the appendix.Though all of the models show good forecasting accuracy, GARCH(1,1) shows the least RMSE and hence this is used for volatility forecasting for pricing European options on Defty.

Option Pricing

Call options are priced on the Defty. The option date is assumed to start on July 31, 2007 and three maturity dates have been considered: 30 day, 60 day and 90 day maturities. The following continuously compounded rates have been assumed in the calculation:

- India: 7% per annum
- US: 4.25% per annum
- Defty: 4.4% per annum

and the options have been priced for various strikes for each maturity.

Monte Carlo Simulation

The Monte Carlo simulations have been carried out using two software packages: Matlab and @Risk with the latter being implemented on Microsoft Excel. The Nifty and exchange rate series were simulated using the relations underlined above and the value of D_T, one each for maturities of 30, 60 and 90 days, calculated for each run. This value was then used to determine the expected value of D_T, $E[D_T]$, by simulating multiple runs; the approximate number of simulations needed for convergence was found to be around 1,00,000. Both Matlab and @Risk yielded similar results thereby acting as double-check for the results.

RESULTS AND CONCLUSION

It is found that the historical volatility over-prices the call option whereas the bi-variate GARCH model shows the least price. The graphs shown in the exhibit below display the variation of call option prices from the three volatility estimates (historical, univariate and bivariate) with different strike values and for three terms of maturity (30, 60 and 90 days).

The authors have modelled the price and volatility processes of the Defty, proving that Defty's price process follows a lognormal distribution and employing both historical and variations of univariate as well as bi-variate GARCH models for volatility estimation. The models were then tested on a 3-month sample and it was found that the GARCH model had higher forecasting power. This model was then used to price options for Defty using Black-Scholes equation. Although the univariate GARCH model does indeed lead to a better pricing than historical estimation, it has been observed that the bi-variate GARCH model pricing is much more efficient and lean. It may be inferred that the pricing mechanism which takes into consideration in randomness in both the exchange rate and Nifty processes yields a superior estimate. Given that both these processes are independent and affect the Defty value, a bi-variate GARCH model that accounts for both concurrent and dynamic dependence in the underlying processes should naturally yield a better pricing model.

APPENDIX

Appendix 1: Defty price Process

$$\frac{dN}{N} = \mu_N dt + \sigma_N dz_N$$

$$\frac{dE}{E} = \mu_E dt + \sigma_E dz_E$$

where N and E are the Nifty and INR/USD exchange rate at time t, and μ_N and μ_E are their drift rates respectively, while σ_N and σ_E are their volatilities.

$$d(NE) = NdE + EdN + dNdE$$

By Ito's Lemma:

$$dNdE = \sigma_N \sigma_E NE(dz_N dz_E) = \sigma_N \sigma_E NE(\rho dt)$$

$$\frac{d(NE)}{NE} = \frac{dE}{E} + \frac{dN}{N} + \rho\sigma_N\sigma_E dt$$

$$= (\mu_N + \mu_E + \rho\sigma_N\sigma_E)dt + \sigma_N dz_N + \sigma_E dz_E$$

$$= (\mu_N + \mu_E + \rho\sigma_N\sigma_E)dt + (\sigma_N^2 + \sigma_E^2 + 2\rho\sigma_N\sigma_E)dz$$

Since $\frac{dD}{D} = \frac{d(NE)}{NE}$,

We have $\mu_D = \mu_N + \mu_E + \rho\sigma_N\sigma_E$ and $\sigma_D^2 = \sigma_N^2 + \sigma_E^2 + 2\rho\sigma_N\sigma_E$

Appendix 2: Risk-neutral Measure

N: Actual Nifty closing value

E: INR/USD exchange rate

N*: (=NE) dollar denominated Nifty values

q: Dividend return yield on Nifty index

∂_E^{US} = Risk neutral drift rate of E under the US money market risk neutral measure (Q_{US})

∂_N^{IN} = Risk neutral drift rate of N under the Indian money market risk neutral measure

$\partial_{N^*}^{US}$ = Risk neutral drift rate of N^* under the US money market risk neutral measure

∂_N^{IN} = Risk neutral drift rate of N under the US money market risk neutral measure

Since, $\partial_E^{US} = r_{US} - r_{IN}$, $\partial_N^{IN} = r_{IN} - q$, $\partial_{N^*}^{US} = r_{US} - q$

Applying the result obtained in Appendix 1 above, we get:

$$\partial_{N^*}^{US} = \partial_{NE}^{US} = \partial_N^{US} + \partial_E^{US} + \rho\sigma_N\sigma_E, \text{ which yields}$$

$$\partial_N^{US} = \partial_{N^*}^{US} - \partial_E^{US} - \rho\sigma_N\sigma_E$$

$$= (r_{US} - q) - (r_{US} - r_{IN}) - \rho\sigma_N\sigma_E$$

Therefore, $\partial_N^{US} = (r_{IN} - q) - \rho\sigma_N\sigma_E$

Appendix 3: RMSE under various ARCH processes

MODEL	RMSE
GARCH(1,1)	***0.009015***
PARCH	0.009021
EGARCH	0.041081
TGARCH	0.009077
ETGARCH	0.009109

Appendix 4: Call Option Pricing

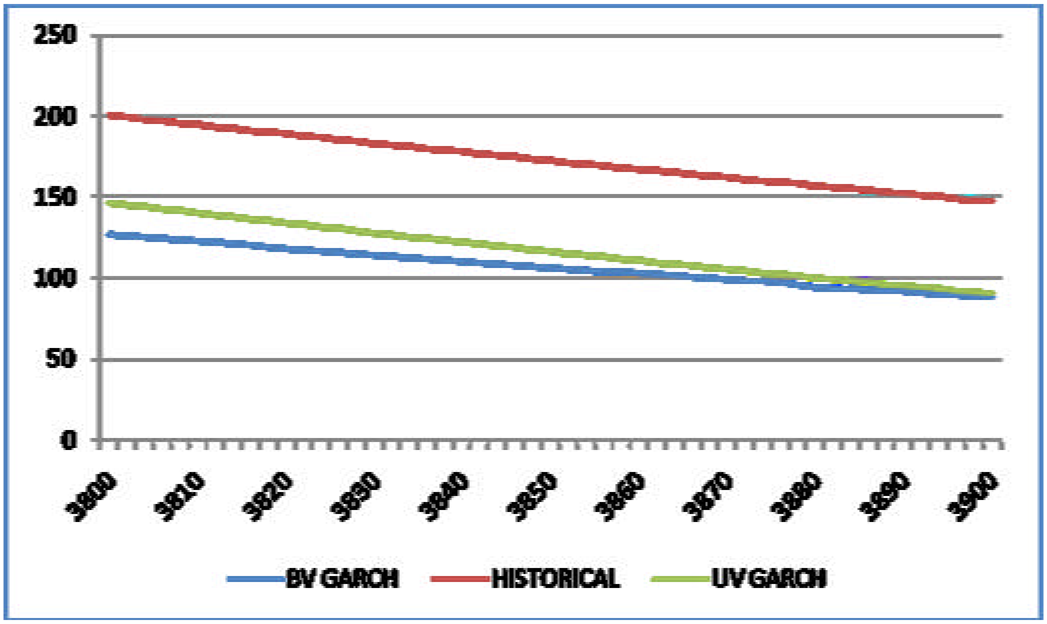

Figure 1: 30 day Call Option Price

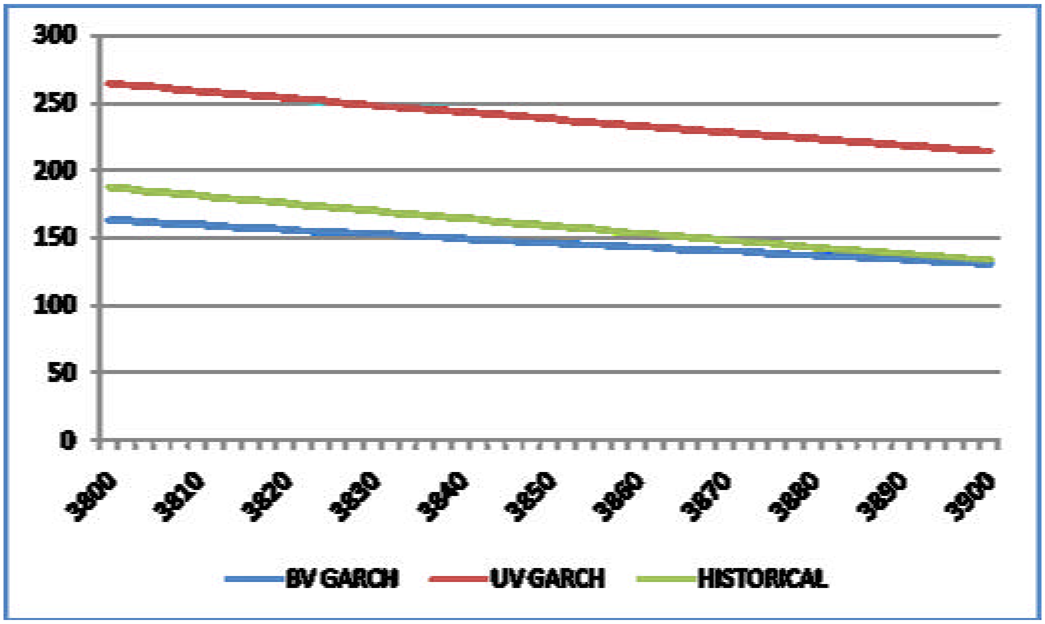

Figure 2: 60 day Call Option Price

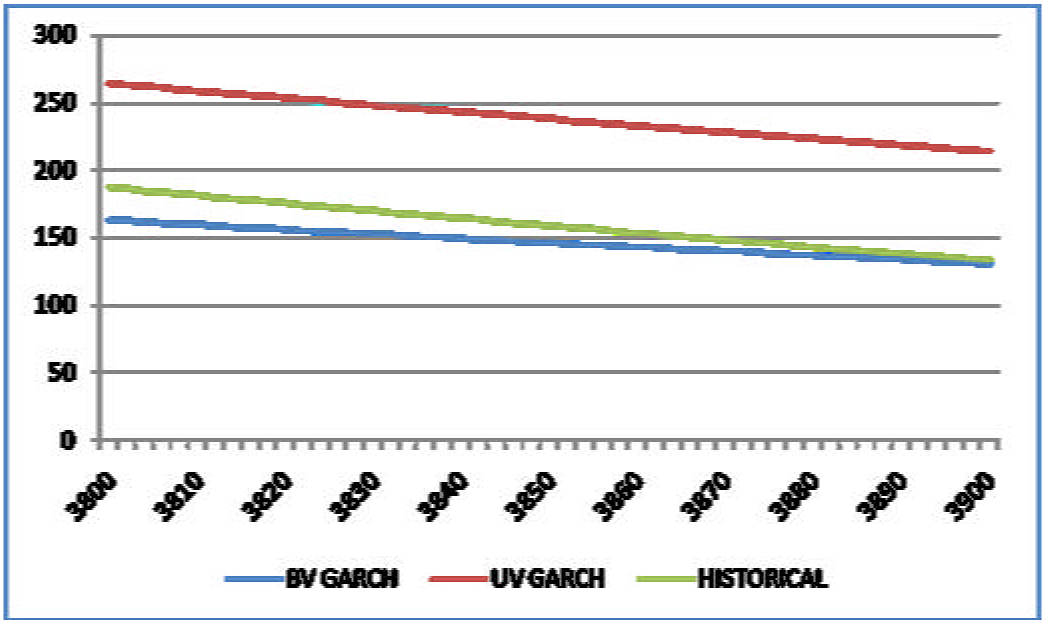

Figure 3: 90 day Call Option Price

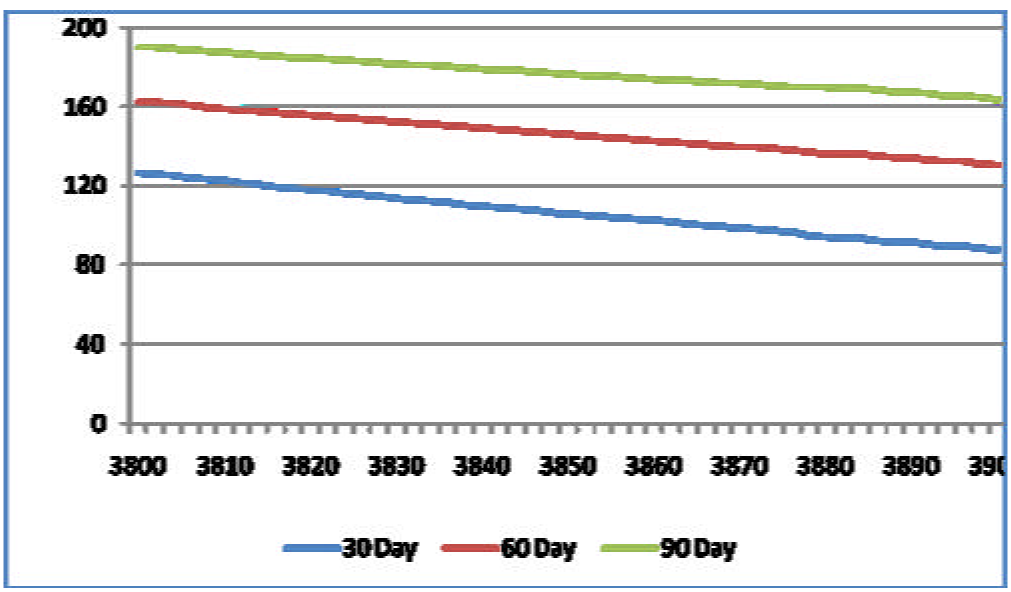

Figure 4: BV GARCH Call Option Prices under with varying maturity

Appendix 5: Put Option Pricing

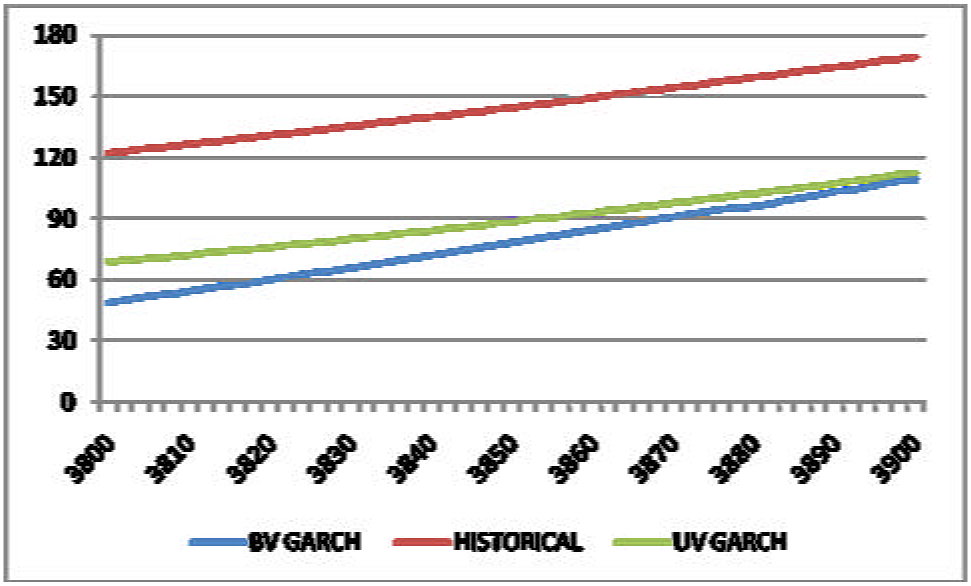

Figure 5: 30 day Put option price

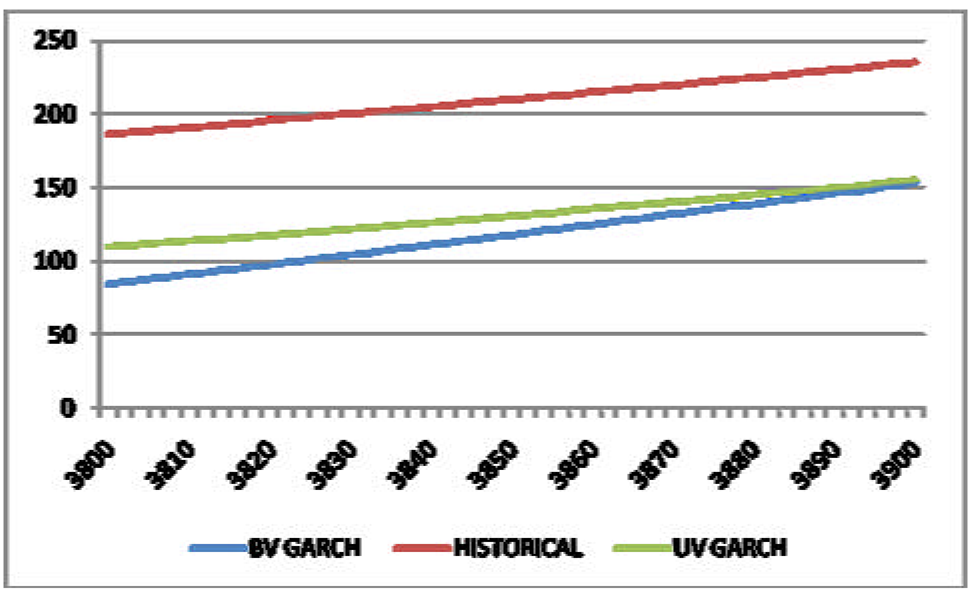

Figure 6: 60 day Put option price

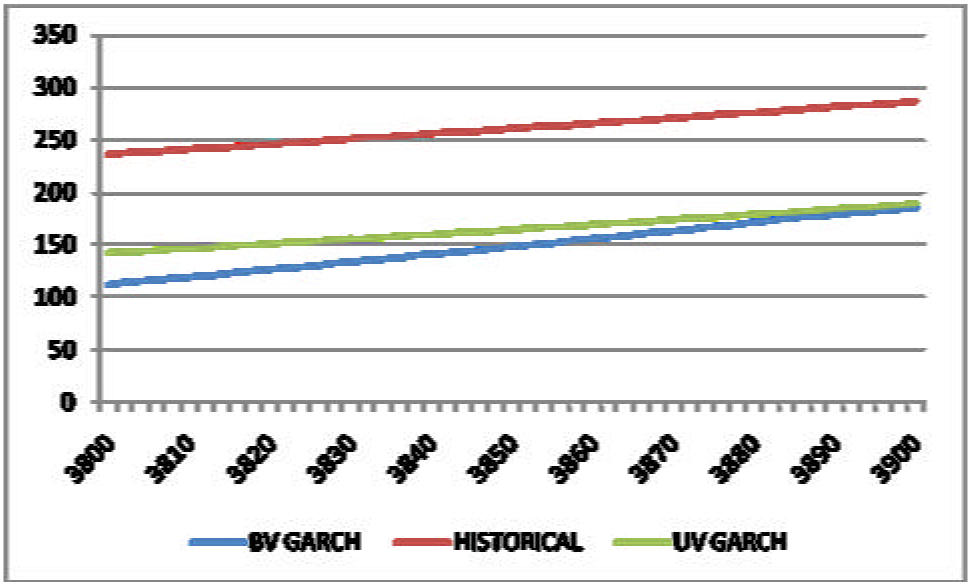

Figure 7: 90 day Put option price

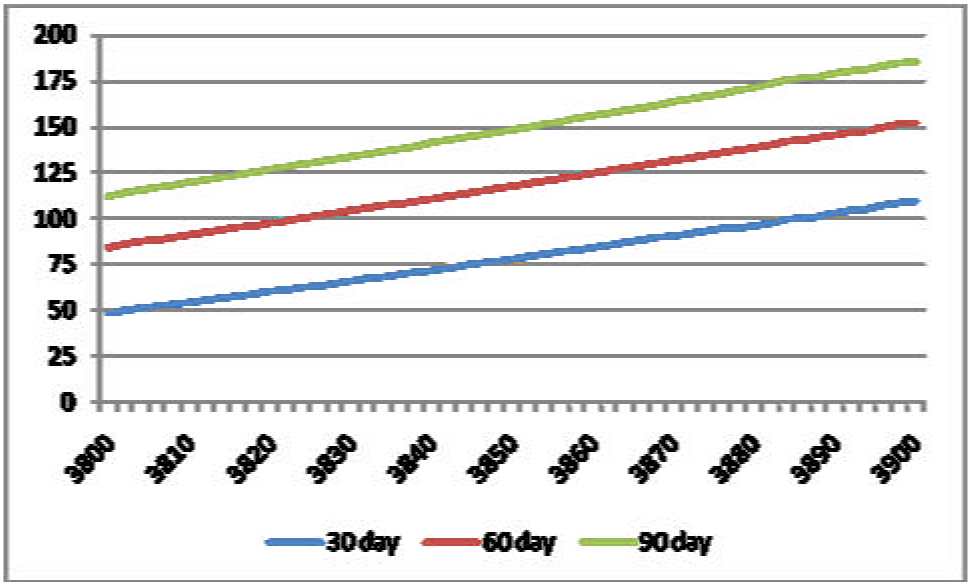

Figure 8: BV GARCH Put Option Prices under with varying maturity

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