Final Proposal 124

Value-At-Risk In Stock And Forex Markets In India: An Application Of The New Transformation-Based Approach

Abstract

We consider a case of VaR analysis when adequately long historical return series for a portfolio/asset is available. Though the primary task here is to estimate the quantile of return distribution, a potential difficulty occurs when returns follow fait-tailed and/or skewed distribution making the simple and convenient normality assumption unrealistic. We also found that the returns in stock and forex markets in India do not follow normal distribution. In order to handle with this non-normality while estimating VaR, we adopted the transformation-based strategy adopted by Samanta (2003). The performance of the transformation-based VaR models is compared with two competing VaR models. Empirical results are quite interesting and identify the transformation-based method quite useful and sensible.

Key Words: Value-at-Risk, Power Transformations to Symmetry/Normality.

JEL Classification: C13, G10

1. INTRODUCTION

The Value-at-Risk (VaR), in recent years, has emerged as an important tool for managing financial risks. Though proposed originally for handling 'market risk', domain of VaR application is soon found much wider and theoretically VaR is applicable even for managing several other financial risks, such as, credit risk, operational risk. In it's role as risk management tool, VaR is useful for several purposes; (a) VaR simply provides a benchmark risk measure, so useful to compare risk involved in different portfolios; (b) VaR is a measure of potential loss from a portfolio; (c) it is also a crucial factor for determining capital charge for market risk exposure (Jorian, 2001; Wilson, 1998).

The VaR, when used for market risk, gives a single number that represents the extent of possible loss from an investment portfolio due to market swings in future. The concept is defined in a probabilistic framework and VaR provides an upper-limit of loss from a portfolio in such a fashion that the instance of actual loss exceeds the VaR during a predefined future time-period has certain fixed/predefined probability. A VaR can also be linked to a confidence level (instead of probability level), which simply indicates the probability that the underlying loss does not exceed the VaR. Also note that confidence level is generally reported in percentage form (i.e. confidence level is derived as 100 multiplied with the probability that the loss remains within VaR). Thus, the relationship between probability level p and confidence level c is described as c=100*(1-p).

Two important terminologies associated with any VaR estimate are the 'holding period' and the 'confidence level'. While the terms 'holding period' refer to the (future) investment horizon, the other terminology is linked to the probability that the portfolio loss would not exceed the VaR number. It is important to note that for a given holding period, VaR number will increase (decrease) with the rise (fall) of confidence level. Similarly, for a given confidence level, VaR has positive association with the holding period – longer is the holding period higher is the VaR. So, the choice of 'confidence level' and 'holding period' would depend on the purpose of estimating the VaR measure.

The VaR for a portfolio can be estimated by analysing the probability distribution of the respective portfolio's return - the VaR is linked to a suitable percentile/quantile of the underlying distribution. So, if the return follows a normal distribution, then using the properties of normal distribution, a percentile can be derived from the corresponding percentile of standard normal distribution (which is readily available from the standard normal distribution table) and mean and standard deviation of the underlying distribution. But in reality, financial market returns seldom follow normal distribution, and the task of estimating VaR has been a challenging one.

It is well recognised that distributions of financial market returns generally possess fatter tails than normal distribution and are skewed. The presence of either thicker tails (excess-kurtosis) or significant skewness or both indicates the non-normality of the underlying distribution. If the specific form of the non-normality were know, one would have easily estimate the VaR from the percentiles of the specific distributional form. But in reality the form of the underlying distribution is not known and one has to discover it from the data. Here one is essentially facing a decision-making problem of selecting one model from many possible alternatives. As well know, the class of non-normal distributions is extremely wide for it includes any possible (continuous) distributions other than normal. The VaR estimation from a mis-specified model may cost havoc to a company and risk managers cover these hazards under what is known as 'model risk' (Christoffersen, et al, 2001). Selection of most accurate model is (perhaps) the best way of minimizing/eliminating the model risk.

The conventional approaches to handle non-normality fall under three broad categories; (i) non-parametric approaches, such as, historical simulation; (ii) fitting suitable non-normal or mixture distribution; (iii) modeling the distribution of extreme return or by modelling only the tails of return distribution. The non-parametric alternatives like historical simulation do not assume any specific form of the return distribution and is quite robust over distributional forms. Besides, these techniques are easy to understand and implement. But this approach suffers from the lack of analytical flexibility and several other disadvantages of what non-parametric approaches share. Alternatively, one can simply fit the parametric form of a suitable non-normal distribution to the observed returns. The class of distributional forms considered would be quite wide including, say, t-distribution, mixture of two normal distribution, hyperbolic distribution, Laplace distribution or so forth, (van den Goorbergh and Vlaar, 1999; Bauer 2000; Linden, 2001). The non-normality, particularly the excess-kurtosis problem can also be captured through a class of conditional heteroscedasticity models The third category, which is also parametric, takes help of extreme value theory and models either the distribution of maximum/minimum return or only the tails of return distribution. The parametric approaches are extremely useful for analytical purpose but identification of actual/appropriate parametric form is extremely difficult.

Another sensible strategy to deal with non-normality while estimating VaR, as proposed recently by Samanta (2003), would involves transforming the (non-normal) return to a (near) normal variable (hence forth we call this as "transformation-based strategy"). Once portfolio returns are transformed into normal variates, one would first derive the suitable percentile for the transformed return distribution, which by construction follows a normal distribution. Applying the properties of the normal distribution, this task is easy. Finally this percentile (for transformed series) can be inverted (by applying the inverse transformation) to derive the percentile of the original return. Samanta (2003) shows that the empirical application of the new strategy to returns from selected stock price index in USA and India provides quite encouraging results. In this study we made an attempt to examine the suitability of the transformation-based approach of VaR for Stock and foreign exchange (FOREX) markets in an emerging market, viz., India. The organisation of the rest of the study is as follows. In Section 2, a brief description of VaR as well as techniques for evaluating accuracy of VaR estimates is given. Section 3 deals with the new transformation-based strategy. Section 4 presents the empirical results. Finally, Section 5 concludes.

2. THE CONCEPT OF VAR

The VaR is a numerical measure of the amount by which a financial position in a risk category could incur loses due to, say, market swings (market risk) during a given holding period. As mentioned earlier, the measure is defined under a probabilistic framework. If W_t denotes the value of the financial assets in the financial position at time instance t, the change in value of the position from time t to t+k would be $\Delta W_t(k) = (W_{t+k} - W_t)$. At time point t, $x_t = \Delta W_t(k)$ is unknown and can be thought of a random variable. Let, $f(x, \beta)$ denotes the probability density function of x_t, β being the vector of unknown parameters. As discussed by Tsay (2002), the VaR (at time point t) of a long-position over time horizon k with probability p, i.e. 100*(1-p) percent confidence level, is defined through the identity,

$$\int_{-\infty}^{-\mathrm{VaR}} f(x,\beta)dx = \mathrm{p} \qquad \textbf{..... (1)}$$

The holder of a long financial position suffers a loss when $\Delta W_t(k) < 0$ and the VaR defined in equation (1) will be positive for small p (conventionally p=0.01 or 0.05). In this case, estimation of VaR depends on the left tail of the distribution of $\Delta W_t(k)$. Here VaR signifies maximum loss attached to the probability level p.

In the case of a short financial position, a loss is incurred when $\Delta W_t(k) > 0$ for underlying assets and for estimating VaR one has to study the right-tail of the distribution of $\Delta W_t(k)$. In particular, the VaR (for time horizon k with probability p) at time t would be determined satisfying the equation.

$$\int_{\mathrm{VaR}}^{\infty} f(x,\beta)dx = \mathrm{p} \qquad \textbf{..... (2)}$$

Thus, for estimating VaR, both left and right tails of the distribution of $\Delta W_t(k)$ are important; the type of financial position (i.e. whether long or short) would indicate the specific tail (i.e. whether left tail or right tail) of the distribution. One may note that the VaR with probability p defined in equation (1) is actually the quantile corresponding to left/lower tail probability p. Similarly, the one defined in equation (2) corresponds to the right-tail probability p and hence to left tail probability (1-p).

2.1 SOME ISSUES WHILE ESTIMATING VAR

In practice, distribution of return (either percentage change or continuously-compounded/log-difference[1]) of the financial position, instead of $\Delta W_t(k)$ defined above, is modeled and thus, the VaR would be estimated based on quantile of the underlying return distribution. If ξ_p denotes the quantile corresponding to left-tail probability p of distribution of k-period percentage change, then k-period VaR for long and short financial positions would be $[(\xi_p/100)W_t]$ and $[(\xi_{1-p}/100)W_t]$, respectively. Alternatively, if ξ_p represents the quantile for log-return (in per cent), then VaR for long and short

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[1] Note that $\Delta W_t(k)$ is the change in value of the assets in the financial position from time point t to (t+k) and the k-period return would be measured by $[100^*\{\Delta W_t(k)/W_t\}]$. Another widely used form of k-period return, known as log-return, is defined by $[100\{\log_e(W_{t+k}) - \log_e(W_t)\}]$. Through out the article, the base of logarithmic transformation is 'e' and therefore, anti-log (i.e. the inverse of log-transformation) of a real number x is anti-log(x) = e^x; sometimes denoted by anti-log(x) = exp(x).

financial positions would be [{exp(ξ_p/100)-1}W_t] and [{exp(ξ_{1-p}/100)-1}W_t], respectively[2]. The multi-period VaR may be derived based on estimated one-period VaR (under certain assumptions).

Sometimes quintiles of return distribution are termed as 'relative VaR' (see for instance, Wong, et al., 2003). On this perception, the VaR for change in value may be termed as 'absolute/nominal VaR'. Thus, the relative VaR using log-return series would be [ξ_p/100] for a 'long position' and [ξ_{1-p}/100] for a 'short position'. Through out this paper, however, we use the single term VaR to indicate either 'nominal VaR' or 'relative VaR', the actual understanding would be made from the contextual meaning.

2.2. AVAILABLE TECHNIQUES FOR ESTIMATING VAR

The central to any VaR measurement strategy has been the estimation of quantiles/percentiles of change in value or return of the portfolio. If the distribution of the change in value or return were normal, then one would have simply estimated the mean and standard deviation of the normal distribution and hence estimate the implied percentiles. But the biggest practical problem of measuring VaR is that the observed return series generally do not follow normal distribution. It is now well recognized that the returns in financial markets follow leptokurtic (fat-tailed) and occasionally skewed distribution. The deviation from normality intensifies the complexity in modelling the distribution of returns and hence estimation of quantiles and VaR. There has been a plethora of techniques to handle non-normality in the context of quantile estimation. Available methodologies can be classified under three broad classes, viz., (i) 'historical simulation', a model-free approach for estimating quantile; (ii) parametric approach for fitting non-normal (fat-tailed and/or skewed) distribution; and (iii) extreme value theory which models either the distribution of extreme observations or tails of underlying distribution. Details of the methods stated above are available in standard books/papers on the topic (see for instances, van den Goorbergh and Vlaar, 1993; Bauer, 2001; Sarma et al, 2003) and for the sake of brevity we do not discuss those here. For a summary of select approaches one may refer to Samanta and Nath (2003).

2.2.1 NORMAL (COVARIANCE) METHOD

The simplest possible VaR method is the normal (covariance) method. If μ and σ are mean and standard deviation, respectively, for return at a future date then VaR would be calculated from the expression (($\mu + \sigma\ z_\alpha$), where z_α represents the percentile corresponding to left-tail probability α of the standard normal distribution and α is the probability level attached to VaR numbers. This

[2] As stated earlier, exp[.] functions in the expressions of VaR are due to the base 'e' chosen for log-transformation.

approach is static in a sense that it models the unconditional return distribution (van den Goorbergh and Vlaar, 1999).

As known, unconditional return distribution generally shows fatter tails (leptokurtosis or excess-kurtosis) than normal, which means that normality assumption to unconditional return distribution may turn out to be unrealistic. It is also known that fatter tails may also be reflection of the changing conditional volatility which can be modelled under suitable simple conditional heteroscedastic models like exponentially weighted moving average used in RiskMetrics (J.P.Morgan/Reuters, 1996) or more advanced models like ARCH, GARCH and so forth (Engle 1982; Bollerslev, 1986; Wong et al., 2003).

2.2.2 METHOD USING TAIL-INDEX

The fat tails of unconditional return distribution can also be handled through extreme value theory using, say, tail-index, which measures the amount of tail fatness. One can therefore, estimate the tail-index and measure VaR based on the underlying distribution. The basic premises of this idea stems from the result that the tails of every fat-tailed distribution converge to the tails of Pareto distribution. The upper tail of such a distribution can be modeled as,

$$\text{Prob}[X > x] \approx C^{\alpha}\ |x|^{-\alpha} \quad (\text{i.e. Prob}[X \leq x] \approx 1 - C^{\alpha}\ |x|^{-\alpha});\quad x > C \qquad \text{..... } \mathbf{(3)}$$

Where, C is a threshold above which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

Similarly, lower tail of a fat-tailed distribution can be modeled as

$$\text{Prob}[X > x] \approx 1 - C^{\alpha}\ |x|^{-\alpha} \quad (\text{i.e. Prob}[X \leq x] \approx C^{\alpha}\ |x|^{-\alpha});\quad x < C \qquad \text{..... } \mathbf{(4)}$$

Where, C is a threshold below which the Pareto law holds; $|x|$ denotes the absolute value of x and the parameter α is the tail-index.

In practice, observations in upper tail of the return distribution are generally positive and those in lower tail are negative. Thus, both of Eq. (3) and Eq. (4) have importance in VaR measurement. The holder of a short financial position suffers a loss when return is positive and therefore, concentrates on upper-tail of return distribution (i.e. Eq. 3) while calculating his VaR (Tsay, 2002, pp. 258). Similarly, the holder of a long financial position would model the lower-tail of return distribution (i.e. use Eq. 4) as a negative return makes him suffer a loss.

From Eqs (3) and (4), it is clear that the estimation of VaR is crucially dependent on the estimation of tail-index α. There are several methods of estimating tail-index, such as, (i) Hill's (1975) estimator and (ii) the estimator under ordinary least square (OLS) framework suggested by van den Goorbergh (1999). We consider here the widely used Hill's estimator of tail-index. A discussion on how to apply Hill's estimator to measure VaR is given below.

Hill's Estimator

For given threshold C in right-tail, Hill (1975) introduced a maximum likelihood estimator of $\gamma = 1/\alpha$ as

$$\hat{\gamma} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{X_i}{C}\right) \qquad \textbf{….. (5)}$$

where X_i's, i=1,2, …..,n are n observations (exceeding C) from the right-tail of the distribution.

To estimate the parameters for left tail, we simply multiply the observations by –1 and repeat the calculations applicable to right-tail of the distribution.

In practice, however, C is unknown and needs to be estimated. If sample observations come from Pareto distribution, then C would be estimated by the minimum observed value, the minimum order statistic. However, here we are not modeling complete portion of Pareto distribution. We are only dealing with a fat-tailed distribution that has right tail that is approximated by the tail of a Pareto distribution. As a consequence, one has to select a threshold level, say C, above which the Pareto law holds. In practice, Eq. (5) can be evaluated based on order statistics in the right-tail and thus, the selection of the order statistics truncation number assumes importance. In other words, one needs to select the number of extreme observations n to operationalise Eq. (3). Mills (1999, pp. 186) discusses a number of available strategies for selecting n. The method adopted in this paper is due to Phillips, et al. (1996). They suggest that optimal value of n should be one, which minimises the Mean-Square-Error (MSE) of the limiting distribution of $\hat{\gamma}$. To implement this strategy, we need estimates of γ for truncation numbers $n_1 = N^{\delta}$ and $n_2 = N^{\tau}$, where $0 < \delta < 2/3 < \tau < 1$. Let $\hat{\gamma}_j$ be the estimate of γ for $n = n_j$, j=1,2. Then the optimal choice for truncation number is $n = [\lambda\, T^{2/3}]$, where λ is estimated as $\hat{\lambda} = |(\hat{\gamma}_1/\sqrt{2})(T/n_2)(\hat{\gamma}_1 - \hat{\gamma}_2)|^{2/3}$. Phillips et al. (1996) recommended setting δ =0.6 and τ = 0.9 (see Mills, 1999, pp. 186).

Estimating VaR Using Tail Index

Once tail-index α is estimated, the VaR can be estimated as follows (van den Goorbergh and Vlaar, 1999). Let p and q (p < q) be two tail probabilities and x_p and x_q are corresponding quantiles. Then p $\approx C^{\alpha} (x_p)^{-\alpha}$ and q $\approx C^{\alpha} (x_q)^{-\alpha}$ indicating that $x_p \approx x_q (q/p)^{1/\alpha}$. Assuming that the threshold in the left-tail of the return (in per cent) distribution corresponds to the m-th order statistics (in ascending order), the estimate of x_p be

$$\hat{x}_p = R_{(m)} \left(\frac{m}{np} \right)^{\hat{\gamma}} \qquad \text{..... (6)}$$

where $R_{(m)}$ is the m-th order statistics in the ascending order of n observations chosen from tail of the underlying distribution; p is the given probability level for which VaR is being estimated; $\hat{\gamma}$ is the estimate of γ. Knowing the estimated quantile $\hat{x}_p$, one can easily calculate the VaR.

As stated above, the methodology described above estimates tail-index and VaR for right tail of a distribution. To estimate the parameters for left tail, we simply multiply the observations by –1 and repeat the calculations.

3. THE NEW TRANSFORMATION BASED APPROACH TO MEASURING VAR

As seen above, the main difficulty for estimating VaR has been the non-normality of return distribution. The existing literature has handled the problem directly in the sense that the non-normal characteristic of return is directly modelled through several alternatives, such as, Historical Simulation (non-parametric), fitting observed return distribution with some non-normal class of distribution (such as, t-distribution, mixture distribution), or modelling tails behaviour of extreme observations (for a review of these methods, see for instance, van den Goorbergh and Vlaar, 1993; Samanta and Nath, 2003). In a recent study Samanta (2003) proposed a new transformation based approach. He also got quite encouraging results for two stock price index portfolios. The new transformation-based approach proposed by Samanta (2003) is presented below;

3.1. BASIC PREMISES

If the underlying variable (say, change in portfolio value or return) r_t is not normally distributed, let there exists a one-to-one continuous function of r_t, say $g(r_t,\theta)$, θ being a constant parameter, which follows a normal distribution. The function form of g(.) would take various different forms, as available in the literature. Because $g(r_t,\theta)$ is a normal variable, its mean and standard deviation can be estimated easily based on the sample observations, provided θ is given. In reality, however, θ is unknown and thus needs to be estimated from the observed data. Let μ_g and σ_g represent the estimated mean and standard deviation of g, respectively. As g(.) represents a one-to-one continuous function, for any real valued number γ we have the events $\{ g(r_t,\theta) < \gamma \}$ and $\{ r_t < g^{-1}(\gamma, \theta) \}$ are equivalent in the sense of probability. In other words, following identity with respect to probability measure holds,

$$\text{Prob}[\, g(r_t,\theta) < \gamma \,] = \text{Prob}[r_t < g^{-1}(\gamma,\theta)] \qquad \textbf{.....(7)}$$

Where Prob(.) denotes the probability measure.

By replacing γ in identity (7) with the p-th quantile of the distribution of $g(r_t,\theta)$, say ν_p, we get the p-th quantile of the unknown distribution of r_t as $\xi_p=g^{-1}(\nu_p,\theta)]$. As $g(r_t,\theta)$ follows a normal distribution, its quantiles are simply $\{\mu_g + \tau_p \sigma_g\}$, where τ_p is the p-th quantile of standard normal distribution. We know that $\tau_{0.01} = -2.33$ and $\tau_{0.05} = -1.65$. As the standard normal distribution is symmetric about zero, the values of $\tau_{0.99}$ and $\tau_{0.95}$ are 2.33 and 1.65, respectively. Now, given the market value of the portfolio and estimated quantiles of underlying return distribution, VaR can be estimated easily.

The idea stated above is intuitively appealing and also easy to understand. But we need to know the functional form of g(.) and also to estimate the unknown transformation parameter θ. The literature on the families of transformations to normality/symmetry comes to the rescue.

3.2. Transformations of a Random Variable to Normality

The attempt towards transforming a random variable x to normality dates back at least to the work of Box and Cox (1964). Thereafter, several other classes of transformations to normality have been proposed in the literature. Some of the useful transformations for our purpose would be the signed power transformation (see, for instance, Bickel and Doksum, 1981), the modulus transformation of John and Draper (1980) and the more recent transformation class offered by Yeo and Johnson (2000).

The signed power transformation to convert a random variable to normal one has the following general form

$$g^{SP}(x,\nu) = \text{sign}(x)\ \{|x|^{\nu} -1 \}\ /\nu, \quad \nu > 0 \qquad \textbf{..... (8)}$$

Where sign(x) and $|x|$ are sign and absolute value of x, respectively and ν is the transformation parameter needs to be estimated from the data on x.

For transforming a symmetric distribution to near normality, John and Draper (1980) suggested the modulus transformation

$$g^{JD}(x,\delta) = \begin{cases} sign(x)\{(1+|x|)^{\delta}-1\}/\delta & \text{if} \delta \neq 0 \\ sign(x)\log(1+|x|) & \text{if} \delta = 0 \end{cases} \quad \text{..... (9)}$$

As per the existing literature, it appears that both, $g^{SP}(x,\nu)$ and $g^{JD}(x,\delta)$ are good to handle kurtosis problem. But these transformations have serious drawbacks when applied to skewed distribution. Particularly, if the distribution of x is mixture of standard normal and gamma densities, then the distributions of both $g^{SP}(x,\nu)$ and $g^{JD}(x,\delta)$ are bimodal and look far away from normal. Besides, in the case of $g^{SP}(x,\nu)$, likelihood function is undefined when some observations of x are zero (Burbidge, et al, 1988). To circumvent with these problems, Yeo and Johnson (2000) proposed following new family of transformations

$$g^{YJ}(x,\lambda) = \begin{cases} \{(1+x)^{\lambda}-1\}/\lambda & if \quad x \geq 0, \lambda \neq 0 \\ \log(1+x) & if \quad x \geq 0, \lambda = 0 \\ -\{(1-x)^{2-\lambda}-1\}/(2-\lambda) & if \quad x < 0, \lambda \neq 2 \\ -\log(1-x) & if \quad x < 0, \lambda = 2 \end{cases} \quad \text{..... (10)}$$

The parameter λ of $g^{YJ}(x,\lambda)$ can be estimated by maximum-likelihood technique (Yeo and Johnson, 2000).

3.3. SELECTION OF TRANSFORMATION TO NORMALITY

We come across several alternative families of transformations to convert a non-normal variable to a near-normal variable. Thus, any exercise on application of normality transformation faces the problem of selecting appropriate family of transformation from various competing classes. Though, theoretical answer to this issue is not very clear, the basic features of each family of transformation discussed above definitely provide certain useful clue. Particularly, three important points are noticed from above; First, to convert a symmetric (or near symmetric) distribution to normality, family of transformation $g^{JD}(x,\delta)$) is useful. The transformation $g^{SP}(x,\nu)$ is not of much use in our case mainly for its limitation in handling zero observations in likelihood function. Second, to convert a skewed distribution to symmetry $g^{YJ}(x,\lambda)$ may be used. Third, if a distribution is non-normal due to both skewness and kurtosis problems, the theory on appropriate choice of power transformation is not clear. In such a scenario, a heuristic approach would suggest to derive first $g^{YJ}(x,\lambda)$ to achieve symmetry and then to apply $g^{JD}(x,\lambda)$ on the already transformed near-symmetric distribution. One, however, need to study the properties of parameter estimates under such a case.

3.4. IMPLEMENTATION OF THE TRANSFORMATION-BASED APPROACH

While implementing the transformation-based approach of VaR estimation, we first need to know whether the underlying distribution is normal or not. If the actual distribution is normal, estimation of VaR would be done by simple normality based techniques. Thus, a test for normality should precede any attempt to adopting transformation-based approach. It is also known that a departure from normality may take place for three possible reasons, (i) non-zero measure of skewness; (ii) deviation of measure of kurtosis from 3 (i.e. excess kurtosis is zero); (iii) both of previous two reasons. Denoting β_1 and β_2 as measures of skewness and excess kurtosis[3], respectively, we have following three hypotheses in this regard.

(i) H_{01}: $(\beta_1,\beta_2)=(0,0)$, which will be tested against the alternative hypothesis H_{11}: $(\beta_1,\beta_2)\neq (0,0)$.

(ii) H_{02}: $\beta_1 =0$, which will be tested against the alternative hypothesis H_{12}: $\beta_1 \neq 0$

(iii) H_{03}: $\beta_2 =0$, which will be tested against the alternative hypothesis H_{13}: $\beta_2 \neq 0$

In our study, we tested the null hypothesis H_{01} by using the Jarque and Bera (1987) test statistics $Q = n[(b_1)^2/6 + (b_2)^2/24]$, where b_1 and b_2 are sample estimates of β_1 and β_2, respectively and n is the number of observation used to derive the said estimates. Under normality, Q is asymptotically χ^2 variable with 2 degrees of freedom. Also note that under normality, each of b_1 and b_2 is also asymptotically normally distributed with mean zero and respective variances 6/n and 24/n implying that each of $[n\ (b_1)^2/6]$ and $[n\ (b_2)^2/24]$ is asymptotically χ^2 variable with 1 degree of freedom (See Gujarati, 1995 for a discussion on Jarque-Bera (1987) test of normality and related issues).

With this background, the proposed transformation-based VaR modelling approach can be implemented through following steps.

- **Step 1:** Test the return series for normality. If normality is accepted, the estimation of VaR will depends upon the quantiles of normal distribution. Otherwise, normality may be rejected for any of the three possible cases of measures of skewness (β_1) and excess-kurtosis

[3] The measure of skewness $\beta_1 = \mu_3/\mu_2^{(3/2)}$ and measure of kurtosis $= \mu_4/\mu_2^2$ indicating that the excess kurtosis $\beta_2 = \mu_4/\mu_2^2 - 3$, where μ_j denotes the j-th order central moment, $j \geq 2$. For normal distribution, $\beta_1 = \beta_2 = 0$.

(β_2), viz. Case (i) $\beta_1 \neq 0$ and $\beta_2 = 0$; Case (ii) $\beta_1 = 0$ and $\beta_2 \neq 0$, and Case (iii) $\beta_1 \neq 0$ and $\beta_2 \neq 0$.

- **Step 2:** If normality is rejected for Case (i) of Step 1, then apply $g^{YJ}(x,\lambda)$ transformation for suitably chosen λ. A standard practice is to estimate λ via a grid-search method by maximizing log-likelihood function over a set of potential alternatives of λ. One may also select λ by minimizing the magnitude of measure of skewness. If normality is rejected for Case (ii) of Step 1, then apply $g^{JD}(x,\delta)$ transformation. The parameter δ may be estimated either by maximizing log-likelihood function or by minimizing extent of excess kurtosis over a set of potential alternative. If normality is rejected for Case (iii), we may proceed via two phases; first apply $g^{YJ}(x,\lambda)$ on the original variable and then pass the transformed variable so obtained through $g^{JD}(x,\delta)$ transformation.
- **Step 3:** Estimate the mean and standard deviation of the near-normal transformed variable in step 2. Using these statistics and the known quantiles of the standard normal distribution, quantiles of the transformed near-normal distribution are estimated.
- **Step 4:** Apply inverse transformation on these quantile to derive the quantiles of the underlying return distribution.
- **Step 5:** Derive VaR using current portfolio value and estimates of quantiles of return distribution.

4. An Application to Stock and FOREX Markets Data in India

In order to demonstrate the possible gain from the new transformation-based VaR modelling approach, it is proposed to assess the performance of proposed approach vis-à-vis select other available VaR models. As stated earlier, the proposed empirical study focuses on the Forex market in India. The database and strategy of empirical analysis are briefly stated below;

4.1. Data

The data used in this study are the daily stock price indices and exchange rates of Indian Rupee. For VaR are calculated at portfolio level, it would be interesting to examine the accuracy of different VaR model with respect to certain stock portfolio. However, composition/components of portfolio differs investors to investors, and it is extremely difficult to suggest any portfolio, which is optimal to all investors. But at the same time, it is neither practical nor manageable to consider all possible stocks portfolio as there would be a large number of individual securities and possible number of portfolios would be astronomically large. A somewhat useful strategy, as adopted by many other researchers also (Bauer, 2000; Christoffersen, et al, 2001; Sarma, et al, 2003) would be to examine the VaR models with respect to certain stock indices, which by construction, represent well-diversified stock portfolio. On this understanding, we consider daily data on three stock price indices (closing price) published by the National Stock Exchange of India Limited (NSEIL), viz, (i) S & P CNX Nifty; (ii) CNX Nifty Junior; and (iii) S & P CNX Defty, for the period from April 1, 1999 to March 31, 2005 (which gives 1509 daily observations on each stock price index considered[4]).

In regards to exchange rate, we donot have readily available indices/portfolio with daily frequency. Though Real or Nominal Effective Exchange Rates (REER or NEER) are compiled based on several foreign currencies and could be considered to track behaviour of certain typical portfolio of foreign currencies, use of these series for VaR analysis is limited for the low data frequency – the RRER and NEER in India are compiled at monthly frequency (instead of daily frequency). So, we consider daily data on spot rate of major four world currencies, viz., US Dollar, British Pound Sterling, Euro Currency and Japanese Yen, in terms of Indian Rupee. Though we do recognize that in reality currency portfolio may include multiple foreign currencies, one still may argue that each currency alone represents a typical portfolio of single currency. The idea of validating VaR models with respect to exchange rates for single foreign currency is also not an exception in the literature for many earlier studies also followed the same path (see for instance, Bauer, 2000). Database for forex market covers the period from April 5, 1999 to March 31, 2005 and contains 1462 daily observations on each exchange rate[5].

4.2. COMPETING VAR MODELS

We propose to assess the performance of the transformation-based VaR model vis-à-vis couple of widely used techniques, such as, Normal (variance-covariance) method and the extreme-value approach using Hill's Estimator (Hill, 1975). All these methods are applied for univariate series on portfolio returns. Also the models we considered are static in a sense that we donot model conditional variance of returns. As known, generally observed leptokurtosis (excess-kurtosis) behaviour of unconditional returns could be due to presence of changing conditional volatility which could be modelled under suitable simple conditional heteroscedastic models like exponentially weighted moving average used in RiskMetrics (J.P.Morgan/Reuters, 1996) or more advanced models

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[4] *Data on Stock price indices are collected from the website of the National Stock Exchange of India Limited (www.nse-india.com).*

[5] *The exchange rate data are collected from various publications of the Reserve Bank of India (RBI). Also see RBI's website (www.rbi.org.in).*

like ARCH, GARCH and so forth (Engle 1982; Bollerslev, 1986; Wong et al., 2003). In our empirical exercise we handle leptokurtotic phenomenon of return distribution, if any, by modelling directly the unconditional distribution of return[6]. So, competing VaR models in our analysis are (i) Normal (covariance) method; (ii) Tail-index based method using Hill's estimator; and (iii) the new transformation-based method.

4.3. STRATEGIES TO EVALUATE VAR MODELS

The accuracy of VaR estimates obtained from competing VaR models would be assessed under several frameworks, such as, (i) backtesting; (ii) Kupieck's test; (iii) loss-function based evaluation of VaR estimates.

Basle Committee Guidelines - Backtesting

As recommended by Basle Committee, central banks do not specify any VaR model, which should be used by their supervised banks. Rather 'internal model approach' is suggested wherein banks are allowed to adopt their own VaR model. There is an interesting issue here. As known, VaR is being used for determining the capital charge – larger the value of VaR, larger is the capital charge. Because larger capital charge implies less profit, banks may have an inclination towards adopting a model that produce lower VaR estimate since that helps to reduce their capital charge. In order to eliminate such inertia of supervised banks, Basle Committee has set out certain requirements on VaR models used by banks to ensure their reliability (Basel Committee, 1996a,b) as follows;

(1) 1-day and 10-day VaRs must be estimated based on the daily data of at least one year
(2) Capital charge is equal to three times the 60-day moving average of 1% 10-day VaRs, or 1% 10-day VaR on the current day, which ever is higher. The multiplying factor (here 3) is known as 'capital multiplier'.

Further, Basle Committee (1996b) provides following Backtesting criteria for an internal VaR model (see van den Goorbergh and Vlaar, 1999; Wong et al., 2003, among others)

(1) One-day VaRs are compared with actual one-day trading outcomes.
(2) One-day VaRs are required to be correct on 99% of backtesting days. There should be at least 250 days (around one year) for backtesting.

[6] Some studies have reported that application of conditional heteroscedastic models does not necessarily improve the VaR estimation. For example, Wong et al. (2003) present empirical evidence that ARCH- and GARCH-based VaR models consistently fail to meet Basle's backtesting criteria.

(3) A VaR model fails in Backtesting when it provides 5% or more incorrect VaRs.

(4) If a bank provides a VaR model that fails in backtesting, it will have its capital multiplier adjusted upward, thus increasing the amount of capital charges.

For carrying out the Backtesting of a VaR model, realized day-to-day returns of the portfolio are compared to the VaR of the portfolio. The number of days when actual portfolio loss was higher that VaR provides an idea about the accuracy of the VaR model. For a good VaR model, this number would approximately be equal to the 1 per cent (i.e. 100 times of VaR probability) of back-test trading days. If the number of violation (i.e. number of days when loss exceeds VaR) is too high, a penalty is imposed by raising the multiplying factor (which is at least 3), resulting in an extra capital charge. The penalty directives provided by the Basle Committee for 250 back-testing trading days is as follows; multiplying factor remains at minimum (i.e. 3) for number of violation upto 4, increases to 3.4 for 5 violations, 3.5 for 6 violations, 3.65 for violations 8, 3.75 for violations 8, 3.85 for violation 9, and reaches at 4.00 for violations above 9 in which case the bank is likely to be obliged to revise its internal model for risk management (van den Goorbergh and Vlaar, 1999).

Statistical Tests of VaR Accuracy

The accuracy of a VaR model can also be assessed statistically by applying Kupiec's (1995) test (see, for example, van den Goorbergh and Vlaar, 1999 for an application of the technique). The idea behind this test is that the VaR-violation (i.e. proportion of cases of actual loss exceeding VaR estimate) should be statistically equal to the probability level for which VaR is estimated. Kupiec (1995) proposed a likelihood ratio statistics for testing the said hypothesis.

If z denotes the number of times the portfolio loss is worse than the true VaR in the sample (of size T, say) then z follows a Binomial distribution with parameters (T, p), where p is the probability level of VaR. Note that here z is actually the summation of I_t at T time points. Ideally, the more z/T closes to p, the more accurate estimated VaR is. Thus the null hypothesis $z/T = p$ may be tested against the alternative hypothesis $z/T \neq p$. The likelihood ratio (LR) statistic for testing the null hypothesis against the alternative hypothesis is

$$LR = 2\left[\log\left(\left(\frac{z}{T}\right)^{z}\left(1-\frac{z}{T}\right)^{T-z}\right) - \log\left(p^{z}(1-p)^{T-z}\right)\right] \qquad \textbf{..... (11)}$$

Under the null hypothesis, LR-statistic follows a χ^2-distribution with 1-degree of freedom.

The VaR estimates are also interval forecasts, which thus, can be evaluated conditionally or unconditionally. While the conditional evaluation considers information available at each time point, the unconditional assessment is made without reference to it. The test proposed by Kupiec provides only an unconditional assessment as it simply counts exceptions (i.e. VaR violations) over the entire backtesting period (Lopez, 1998). In the presence of time-varying volatility, the conditional accuracy of VaR estimates assumes importance. Any interval forecast ignoring such volatility dynamics may have correct unconditional coverage but at any given time, may have incorrect conditional coverage. In such cases, the Kupiec's test has limited use as it may classify inaccurate VaR as acceptably accurate.

Christoffersen (1998) develops a three step testing procedure: a test for correct unconditional coverage (which is same as Kupiec's test), a test for 'independence', and a test for correct 'conditional coverage' (Berkowitz and O'Brien, 2002; Sarma, et al., 2003). All these tests use Likelihood-Ratio (LR) statistics.

Evaluation of VaR Models Using Loss-Function

All the tests mentioned above, ultimately deal with the frequency of the occurrence of VaR violations, either conditional or unconditional, during the backtesting trading days. These tests, however, do not look at the extent/magnitude of additional loss (excess of estimated VaR) at the time of VaR violations/failures. However, a portfolio manager may prefer the case of more frequent but little additional loss than the case of less frequent but huge additional loss. The underlying VaR model in the former case may fail in backtesting but still the total amount of loss (after adjusting for penalty on multiplying factor if any) during the backtesting trading days may be less than that in later case. So long as this is the case, a portfolio manager may even prefer to accept a VaR model even if it fails in backtesting and may be ready to pay penalty (for excess number of VaR violations). This means that the objective function of a portfolio manager is not necessarily be the same as that provided by the backtesting. Each manager may set his own objective function and try to optimize that while managing market risk. But, loss-functions of individual portfolio managers are not available in public domain and thus, it would be impossible to select a VaR model appropriate for all managers. However, discussion on a systematic VaR selection framework by considering a few specific forms of loss-function would provide insight into the issue so as to help individual manager to select a VaR model on the basis of his own loss-function. On this perception, it would be interesting to illustrate the VaR selection framework with the help of some specific forms of loss-function.

The idea of using loss-function for selecting VaR model, perhaps, is proposed first by Lopez (1998). He shows that the binomial distribution-based test is actually minimizing a typical loss-function – gives score 1 for a VaR exception and a score 0 otherwise. In other words, the implied loss-function in backtesting would be an indicator function I_t which assumes a value 1 at time t if the loss at t exceeds corresponding VaR estimate and assumes a value zero othersise. However, it is hard to imagine an economic agent who has such a utility function: one which is neutral to all times with no VaR violation and abruptly shifts to score of 1 in the slightest failure and penalizes all failures equally (Sarma, et al., 2003). Lopez (1998) also considers a more generalised loss-function which can incorporates the regulatory concerns expressed in the multiplying factor and thus is analogous to the adjustment schedule for the multiplying factor for determining required capital. But, he himself see that, like the simple binomial distribution-based loss-function, this loss-function is also based on only the number of exceptions (VaR violations) in backtesting observations – with paying no attention to another concern, the magnitudes of loss at the time of failures. In order to handle this situation, Lopez (1998) also proposes a different loss-function addressing the magnitude of exception as follows;

$$L_t = \begin{cases} 1 + (Loss_t - VaR_{t-1})^2 & if \;\; Loss_t > VaR_{t-1} \\ 0 & otherwise \end{cases} \qquad \textbf{..... (12)}$$

where $Loss_t$ and VaR_t, respectively, are the magnitude/amount of loss and estimated Value-at-Risk at time t. L_t denotes the score in loss-function at time t.

In the spirit of Lopez (1998), Sarma et al. (2003) consider two loss-functions, viz., regulatory loss function and the firm's loss function, as follows;

Regulatory Loss Function

$$L_t = \begin{cases} (Loss_t - VaR_{t-1})^2 & if \;\; Loss_t > VaR_{t-1} \\ 0 & otherwise \end{cases} \qquad \textbf{..... (13)}$$

Firm's Loss Function

$$L_t = \begin{cases} (Loss_t - VaR_{t-1})^2 & if \;\; Loss_t > VaR_{t-1} \\ \alpha \;\; VaR_{t-1} & otherwise \end{cases} \qquad \textbf{..... (14)}$$

where α represents the opportunity cost of capital.

4.3. EMPIRICAL RESULTS

The returns we considered are continuously compounded calculated as

R_t=100 *[$\log_e(P_t) - \log_e(P_{t-1})$] **….. (15)**

Were P_t represents stock price index or exchange rate as the case may be for t-th day in the database and R_t denotes corresponding daily continuously compounded return.

We consider VaR in percentage form, which means that VaR number will reflect the maximum percentage loss with given probability and holding period. In other words, estimated VaR corresponds to the possible loss for a portfolio of value 100 units. We shall estimate, on the t-th day, the VaR for the (t+1)-th and future dates. Also we consider only the cases of lower-tail VaRs. As discussed earlier, the task ultimately boils down to estimation of 1st percentile (for VaR with 99% confidence level or equivalently with probability level p=0.01) or 5th percentile (for VaR with probability level p=0.05) of the return distribution. The potential difficulty would arise if the returns do not follow normal distribution.

4.3.1 Empirical Return Distributions and Transformations to Normality

We first examine whether return distributions could be considered as normal. As known, for a normal distribution measures of skewness (β_1) and excess-kurtosis (β_2) are all zero. We performed the null hypotheses H_{01}, H_{02} and H_{03} against the alternative hypotheses discussed earlier. Results of these tests for chosen stock indices are presented in Table 1. Corresponding results for exchange rates are reported in Table 2. In these Tables, we report the observed values of chi-square test statistics and corresponding probability value, denoted by 'p-value'. A null hypothesis would be accepted at conventional 1% (or 5%) level of significance if the p-value of corresponding test statistics exceeds 0.01 (or 0.05). The results presented in Table 1 & 2 show that the p-values for the original return series for all the selected asset/portfolios are substantially lower than 0.01, strong rejection of hull hypotheses at 1% significance level (hence at 5% significance level also). We, therefore, decide to pass each original return series through the normality transformation so as to obtain transformation of returns, which follow (approximately) normal distribution.

TABLE 1: RESULTS FOR TESTING NORMALITY OF RETURNS IN STOCK MARKET

Asset/ Portfolio	Measure of Skewness	χ_1^2 for Skewness (Testing H_{02})	Excess Kurtosis	χ_1^2 for Excess Kurtosis (Testing H_{03})	Jarque-Bera Statistics (Testing H_{01})
Nifty	-0.5706	81.7359** (0.0000)	5.6519	2004.4556** (0.0000)	2086.1915** (0.0000)
Nifty Junior	-0.7894	156.3937**	3.8565	933.2669**	1089.6606**

		(0.0000)		(0.0000)	(0.0000)
Defty	-0.6331	100.6082**	6.3081	2496.9364**	1503.2489**
		(0.0000)		(0.0000)	(0.0000)

Note: Figures within ( ) indicate significance level (i.e. p-value) of corresponding statistics; '**' indicates significant at 1% level of significance.

TABLE 2: RESULTS FOR TESTING NORMALITY OF RETURNS IN FOREX MARKET

Asset/ Portfolio	Measure of Skewness	χ_1^2 for Skewness (Testing H_{02})	Excess Kurtosis	χ_1^2 for Excess Kurtosis (Testing H_{03})	Jarque-Bera Statistics (Testing H_{01})
US Dollar	-0.3010	21.1552**	17.7319	18354.2750**	18375.4302**
		(0.0000)		(0.0000)	(0.0000)
Pound Sterling	-0.0753	1.3231	0.7373	31.7336**	33.0567**
		(0.2500)		(0.0000)	(0.0000)
Euro	-0.0259	0.1568	0.8708	44.2622**	44.4189**
		(0.6921)		(0.0000)	(0.0000)
Japanese Yen	0.0960	2.1517	1.6844	165.6248**	167.7766**
		(0.1424)		(0.0000)	(0.0000)

Note: Figures within ( ) indicate significance level (i.e. p-value) of corresponding statistics; '**' indicates significant at 1% level of significance.

We follow a simple two-step transformation strategy irrespective of the fact of whether the underlying return series is skewed/leptokurtotic (when excess-kurtosis turns out to be significantly different than zero). In the first step, we apply the $g^{YJ}(.,\lambda)$ transformation proposed by Yeo and Johnson (2000) on return, so that the transformed variable have near-symmetric distribution. The transformation parameter λ is estimated through a grid-search over the set of potential alternatives {0, 0.001, 0.002, 0.003, …., 1.999, 2}. The criteria used to choose optimal λ from the grid-search would be 'maximum likelihood function' (Yeo and Johnson, 2000) or heuristically, 'minimum absolute value of skewness measure". In the second step we handle the problem of excess kurtosis. Thus the transformed series (which are near-symmetric) are passed through the $g^{JD}(.,\delta)$ transformation proposed by John and Draper (1980). The parameter δ is estimated via a grid-search over {-2, -1.999, -1.998, ……., 1.999, 2}. As earlier, the criteria for selecting optimal δ would be 'maximum likelihood function' (John and Draper, 1980) or heuristically, 'minimum absolute vale of excess kurtosis measure'. Based on experimentation on our database, we found that while 'absolute skewness/excess-kurtosis based estimates of λ and δ work relatively better for stock indices, the corresponding estimates for exchange rate data are better when 'likelihood-function' criteria is used. The optimal estimates of λ and δ for a return series are chosen accordingly. We hope that the final transformation y= $g^{JD}\{g^{YJ}(r, \hat{\lambda}), \hat{\delta}\}$, where $\hat{\lambda}$ and $\hat{\delta}$ are estimates of λ and δ, respectively for the return r, is a (near) normal variable. In order to verify this position, we test the hypotheses H_{01}, H_{02} and H_{03} (against corresponding alternative hypotheses) for all transformed variables/returns. Corresponding empirical results for 'stock price indices' and 'exchange rate' data are given in Table 3 and Table 4, respectively.

As seen from Table 3 and 4, application of the transformation induced normality for all returns considered except the one corresponding to the spot exchange rate of US Dollar against Indian Rupee. But seeing carefully the Table 2 and Table 4, it appears that though the underlying chi-square test statistics for testing normality are significant (at 1% level) in this worst case, the magnitudes of underlying test statistics values (Table 4) are relatively lower for transformed variable (as compared to those for original return, see Table 2).

TABLE 3: RESULTS FOR TESTING NORMALITY OF TRANSFORMATION OF RETURNS IN STOCK MARKET

Asset/ Portfolio	Transfor-mation Parameters	Measure of Skewness	χ_1^2 for Skewness (Testing H_{02})	Excess Kurtosis	χ_1^2 for Excess Kurtosis (Testing H_{03})	Jarque-Bera Statistics (Testing H_{01})
Nifty	$\hat{\lambda} = 1.088$ $\hat{\delta} = 0.306$	-0.0639	1.0638 (0.3116)	0.0002	0.0000 (0.9986)	1.0239 (0.5993)
Nifty Junior	$\hat{\lambda} = 1.136$ $\hat{\delta} = 0.382$	-0.1116	3.1272 (0.0770)	-0.0001	0.0000 (0.9998)	3.1272 (0.2095)
Defty	$\hat{\lambda} = 1.094$ $\hat{\delta} = 0.308$	-0.0572	0.8221 (0.3645)	-0.0013	0.0000 (0.9921)	0.8223 (0.6623)

Note: *Figures within ( ) indicate significance level (i.e. p-value) of corresponding statistics. All test statistics presented in this table are statistically insignificant at 5% level of significance..*

TABLE 4: RESULTS FOR TESTING NORMALITY OF TRANSFORMATION OF RETURNS IN FOREX MARKET

Asset/ Portfolio	Transfor-mation Parameters	Measure of Skewness	χ_1^2 for Skewness (Testing H_{01})	Excess Kurtosis	χ_1^2 for Excess Kurtosis (Testing H_{03})	Jarque-Bera Statistics (Testing H_{01})
US Dollar	$\hat{\lambda} = 1.075$ $\hat{\delta} = -2.000$	0.1240	3.5904 (0.0581)	2.0822	253.0867** (0.0000)	256.6772** (0.0000)
Pound Sterling	$\hat{\lambda} = 1.048$ $\hat{\delta} = 0.540$	-0.0072	0.0122 (0.9120)	0.0471	0.1273 (0.7191)	0.1415 (0.9317)
Euro	$\hat{\lambda} = 1.005$ $\hat{\delta} = 0.653$	-0.0032	0.0021 (0.9636)	0.1149	0.7710 (0.3800)	0.7731 (0.6794)
Japanese Yen	$\hat{\lambda} = 0.961$ $\hat{\delta} = 0.286$	-0.0204	0.0973 (0.7550)	0.0616	0.2215 (0.6379)	0.3189 (0.8526)

Note: *'**' significant at 1% level of significance.*

4.3.2 ESTIMATION OF VAR

In Table 5 & 6, we present estimated 1-day VaRs at the last day of our database (i.e. March 31, 2005). Also given are the average 1-day VaRs in last 60 days of the database. These VaRs could be used for determining capital charge for the next day (i.e. April 1, 2005). All VaR estimations pertain to the lower-tails of return distributions and are estimated using a rolling window of size 300 days[7]. In other words, VaR for time (t+1) has been calculated at time t based on daily observations for the period from (t-299)-th day to t-th day in the database. For given h-days holding period, the h-days VaR can be calculated approximately as a function of h and 1-day VaR. As per the Basle Accord, capital charge for market risk in a day would be the maximum of (i) average of 10-day 99% VaRs in previous 60 days multiplied by a prescribed number k, known as multiplying factor, and (ii) the 10-day 99% VaR in previous day. From Tables 5 and 6, it is clear that the estimated VaR numbers vary quite substantially across the models indicating that the sensitiveness of capital charge on choice of VaR model.

[7] *As per the guidelines one should calculate 1-day VaR based on at least one-year daily data, i.e. about 250-260 days' data.*

TABLE 5: VAR ESTIMATION (LOWER/LEFT TAIL) FOR STOCK DATA

Asset/ Portfolio	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
Nifty	2.683 (2.679)	2.301 (2.369)	2.526 (2.501)	3.799 (3.810)	4.736 (4.755)	4.966 (4.975)
Nifty Junior	2.996 (2.957)	2.724 (2.574)	2.587 (2.488)	4.253 (4.226)	7.824 (7.684)	5.493 (5.353)
Defty	2.802 (2.766)	2.360 (2.328)	2.675 (2.592)	3.964 (3.938)	4.795 (4.832)	5.223 (5.125)

Note: Figures within ( ) indicates average of one-day VaR in last 60-days in the database.

TABLE 6: VAR ESTIMATION (LOWER/LEFT TAIL) FOR EXCHANGE RATE DATA

Asset/ Foreign Currency	**P=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
US Dollar	0.508 (0.474)	0.463 (0.407)	0.443 (0.398)	0.715 (0.669)	1.301 (1.130)	0.949 (0.813)
Pound Sterling	0.971 (0.983)	0.967 (0.984)	1.018 (1.035)	1.392 (1.411)	1.328 (1.968)	1.336 (1.686)
Euro	1.098 (1.148)	1.073 (1.164)	1.125 (1.189)	1.571 (1.643)	2.119 (2.072)	1.848 (1.964)
Japanese Yen	0.977 (0.966)	0.981 (0.947)	0.980 (0.958)	1.387 (1.379)	1.639 (1.577)	1.615 (1.590)

Note: Figures within ( ) indicates average of one-day VaR in last 60-days in the database

4.3.3 Empirical Evaluation of VaR Estimates/Models

We now assess the accuracy of competing VaR models by passing those through different validation criteria discussed earlier. This assessment will also help us to compare the relative performance of competing VaR models. For any such validation and comparison, the strategy adopted is similar to that of Bauer (2000); we keep latest 1000 days in our database for validating VaR models. The validation begins with the estimation of VaRs at the (T-1000)-th day in our database, where T represents the total number of data/days in the database for underlying return series/portfolio. A rolling sample of 300 days is used for estimating VaR numbers. After estimation, we count how often in the following 10 days actual portfolio loss (i.e. negative of return series) exceeds estimated VaR.

The task is done for two alternative probability levels, viz., 0.01 and 0.05. (i.e. confidence levels of 99% and 95%, respectively). We then shift the estimation period by 10 days into the future and went on. The process of estimation and comparing loss with VaR in following 10 days was repeated 100 times so as to cover all 1000 days in the validation period.

We first present the results of backtesting. While Table 7 reports the percentage of VaR failures for stock indices data, corresponding results for exchange rate data are presented in Table 8. As can be seen, though all VaR models perform reasonably well for 95% VaR, the VaR violation for 99% VaR varies considerably across competing models – a findings similar to Bauer (2000), who compared two competing VaR methods, viz., 'hyperbolic distribution-based method' and 'normal method'. Perhaps, the aberration that occurred in the case of probability level p=0.05 is the case of US Dollar exchange rate (against Indian Rupee) where the proportion of VaR violations is as high as 7.7% (instead of theoretical 5%) for tail-index method. Interestingly, for 99% confidence level, performance of 'transformation-based method', judged by the 'proportion of VaR violation', is generally the best followed by 'tail-index' and 'normal' methods in that order.

TABLE 7: PERCENTAGE OF VAR VIOLATIONS BY COMPETING MODELS FOR STOCK DATA

Asset/ Portfolio	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
Nifty	4.7	5.3	4.6	1.7	1.3	1.3
Nifty Junior	4.6	4.9	4.6	2.0	0.8	1.1
Defty	4.7	5.4	4.5	1.8	1.3	1.1

TABLE 8: FREQUENCY OF VAR VIOLATIONS BY COMPETING MODELS FOR EXCHANGE RATE DATA

Asset/ Foreign Currency	**P=0.05**			**P=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
US Dollar	5.4	7.6	6.6	2.2	2.1	2.3
Pound Sterling	5.4	4.9	5.0	1.7	1.5	1.2
Euro	4.6	4.5	4.5	1.2	1.1	1.0
Japanese Yen	5.5	5.1	5.5	1.3	0.8	0.9

THE KUPIEC'S TEST FURTHER ESTABLISHES THE SUPERIORITY OF THE TRANSFORMATION-BASED METHOD OVER BOTH 'NORMAL' AND 'TAIL-INDEX' METHODS. IT IS INTERESTING TO NOTE FROM TABLE 9 THAT IN THE CASE OF STOCK PRICE DATA, THIS TEST IDENTIFIES ALL THREE COMPETING VAR MODELS STATISTICALLY ACCURATE FOR 95% CONFIDENCE LEVEL

(I.E. WHEN PROBABILITY LEVEL IS FIXED AT P=0.05). THIS WAS ALSO EXPECTED FROM EARLIER FINDINGS VAR VIOLATIONS. BUT WE SEE DRASTIC IMPROVEMENT FOR TRANSFORMATION-BASED METHOD WHEN PROBABILITY LEVEL FOR VAR IS FIXED AT P =0.01. NOW, THE KUPIEC'S TEST REJECTS THE ACCURACY OF VAR ESTIMATES USING NORMAL METHOD FOR ALL THREE STOCK PRICE INDICES CONSIDERED. THE ESTIMATES OF VAR USING TAIL-INDEX AND TRANSFORMATION-BASED METHODS ARE NOW STATISTICALLY ACCURATE BUT BETTER FOR THE LATER (AS THE P-VALUE OF THE LR TEST-STATISTICS IS HIGHER OR EQUAL TO THOSE FOR TAIL-INDEX METHOD).

In the case of exchange rate data also, the Kupiec's test identifies the 'transformation-based methods' as the best in all cases except for US Dollar (Table 10). In some cases, normal method also produces quite accurate VaR estimates, but even then, transformation-based method perform further better. For example consider the case of 'euro' or 'Japanese Yen' and probability level 0.01. Here, VaR violation from normal method is statistically consistent with the theoretical probability 0.01 as corresponding Kupiec's Chi-square statistics are statistically insignificant at 5% level of significance. So is the case with the Chi-Square statistics for other two competing VaR models. But the interesting point to note here is that even in such case, the value of the Chi-square statistics is the minimum for 'transformation-based' method.

TABLE 9: RESULTS OF KUPIEC'S TESTS FOR STOCK DATA

Asset/ Portfolio	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
Nifty	0.1932 (0.6603)	0.1860 (0.6663)	0.3457 (0.5566)	4.0910* (0.0431)	0.8306 (0.3621)	0.8306 (0.3621)
Nifty Junior	0.3457 (0.5566)	0.0212 (0.8843)	0.3457 (0.5566)	7.8272** (0.0052)	0.4337 (0.5102)	0.0978 (0.7544)
Defty	0.1932 (0.6603)	0.3287 (0.5664)	0.5438 (0.4608)	5.2251** (0.0223)	0.8366 (0.3621)	0.8306 (0.3621)

Note: Figures within ( ) denote p-value (probability values); '*' and '**' denote significant at 5% and 1% level of significance, respectively.

TABLE 10: RESULTS OF KUPIEC'S TESTS FOR EXCHANGE RATE DATA

Asset/ Foreign Currency	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
US Dollar	0.3287	12.3621**	4.9184*	10.8382**	9.2840**	12.4853**
	(0.5664)	(0.0004)	(0.0266)	(0.0010)	(0.0023)	(0.0004)
Pound Sterling	0.3287	0.0212	0.0000	4.0910*	2.1892	0.3798
	(0.5664)	(0.8843)	(1.0000)	(0.0431)	(0.1390)	(0.5377)
Euro	0.3457	0.5438	0.5438	0.3798	0.0978	0.0000
	(0.5565)	(0.4608)	(0.4608)	(0.5377)	(0.7544)	(1.0000)
Japanese Yen	0.5108	0.0209	0.5105	0.8306	0.4337	0.1045
	(0.4749)	(0.8850)	(0.4749)	(0.3621)	(0.5102)	(0.7465)

Note: Figures within ( ) denote p-value (probability values); '*' and '**' denote significant at 5% and 1% level of significance, respectively.

We now discuss the values of Regulators' loss-function (given in Eq. 13) over the validation dataset. Relevant results, given in Table 11 (for stock data) and Table 12 (for exchange rate data), show that the transformation-based VaR model outperforms other two competing models for stock data, though in case of exchange rate data results are mixed. Interestingly, transformation-based model performs better than normal method in most of the cases (except only for two cases, viz., for US Dollar and Japanese Yen, both corresponding to p=0.05). As known, these regulators' function penalize a model depending upon the extent of loss excess of estimated VaR – leaving the very fact of instances of VaR violation contributing to the penalty/loss function.

TABLE 11: RESULTS OF REGULATOR'S LOSS-FUNCTION FOR STOCK DATA

Asset/ Portfolio	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
Nifty	219.31	215.44	213.69	140.84	117.07	100.51
Nifty Junior	310.13	299.97	302.99	203.26	150.04	146.61
Defty	246.68	239.43	239.39	164.94	131.89	119.98

TABLE 12: RESULTS OF REGULATORS'S LOSS-FUNCTION FOR EXCHANGE RATE DATA

Asset/ Foreign Currency	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
US Dollar	5.31	6.56	6.02	3.96	2.82	3.71
Pound Sterling	9.82	9.08	9.53	2.44	1.27	1.58
Euro	10.32	9.95	10.22	2.63	2.09	2.02
Japanese Yen	15.57	16.17	16.13	6.29	5.57	4.80

The Lopez's loss-function (Eq. 12) on the other hand imposes a penalty on a model depending upon the frequency of VaR violation and also on the extent of excess loss (excess over VaR). From this point of view this loss-function appears more general than the regulators' loss function. We present estimated values of Lopez's loss-function over validation period in Table 13 (stock indices data) and Table 14 (exchange rate data). Based on these Tables also it is clear that the transformation-based VaR model outperforms other two competing models for 'stock data' (Table 13) and also majority of cases for exchange rate data (Table 14). Based on these results it is well established that the 'transformation-based' method is at least a sensible alternative for VaR modelling.

TABLE 13: RESULTS OF REGULATORS LOSS-FUNCTION FOR STOCK DATA

Asset/ Portfolio	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
Nifty	266.31	268.44	259.69	157.84	130.07	113.51
Nifty Junior	356.13	348.97	348.99	223.26	158.04	157.61
Defty	293.68	293.43	284.39	182.94	144.89	131.98

TABLE 14: RESULTS OF LOPEZ'S LOSS-FUNCTION FOR EXCHANGE RATE DATA

Asset/ Foreign Currency	**p=0.05**			**p=0.01**		
	Normal	**Tail-Index**	**Trans. Based**	**Normal**	**Tail-Index**	**Trans. Based**
US Dollar	59.13	82.56	72.02	25.96	23.82	26.71
Pound Sterling	63.82	58.08	59.53	19.44	16.27	13.58
Euro	56.33	54.95	55.22	14.63	13.09	12.02
Japanese Yen	70.57	67.17	71.13	19.29	13.57	13.80

5. CONCLUDING REMARKS

We consider a case of estimating VaR when adequately long history of returns from a portfolio is available. If returns were normally distributed, estimation of VaR would be made simply by using first two moments of the distribution and the tabulated values of standard normal distribution. But the experience from empirical literature shows that the task is potentially difficult for the fact that the return distribution seldom follows normal distribution. It is observed that empirical return distributions have thicker tails than normal and also at times are skewed. In order to handle the non-normality, a number of techniques have been proposed in the literature. Recently, Samanta (2003) proposed a new strategy based on transformations to normality. He argues that a return series (which possibly does not follow normal distribution) may first of all be transformed to a (near) normal variable by applying suitable transformations to normality/symmetry; required quantiles of this near-normal transformed distribution would be estimated, and finally the value of the inverse function of normality transformation at the estimated quantiles would produce required quantiles for the original return and hence VaR for actual portfolio. Logically, the performance of proposed strategy depends upon the efficiency of the applied transformation to convert a non-normal distribution to a (near) normal distribution. Unlike this, the efficiency of conventional strategies lie in their capability in approximating unknown (true) distribution of portfolio return. The performance of new VaR modelling strategy has been assessed with respect to select stock price indices and exchange rates for Indian financial markets. The empirical results are quite encouraging and support the usefulness of the new VaR modelling strategy.

References

1. Basle Committee (1996a), ***Amendment to the Capital Accord to Incorporate Market Risks***, Bank for International Settlements.
2. Basle Committee (1996b), ***Supervisory Framework for the Use of 'Backtesting' in Conjunction with Internal Models Approach to Market Risk*****,** Bank for International Settlements.
3. Bauer, Christian (2000), "Value at Risk Using Hyperbolic Distributions", ***Journal of Economics and Business***, Vol. 52, pp. 455-67.
4. Berkowitz, Jeremy and James O'Brien (2002), "How Accurate are Value-at-Risk Models at Commercial Banks?", ***Journal of Finance***, Vol. LVII, No. 3, June, pp. 1093-111.
5. Bickel, P.J. and K.A. Doksum (1981), "An Analysis of Transformations Revisited", ***Journal of American Statistical Association***, vol. 76, pp. 296-311.
6. Billio, Monica and Loriana Pelizzon (2000), "Value-at-Risk: A Multivariate Switching Regime Approach", ***Journal of Empirical Finance***, Vol. 7, pp. 531-54.
7. Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity", ***Journal of Econometrics***, Vol. 31, pp. 307-27.

8. Box, G.E.P. and D.R. Cox (1964), "An Analysis of Transformations" (with Discussion), ***Journal of Royal Statistical Association***, Vol. 76, pp. 296-311.
9. Burbidge John B., Lonnie Magee and A. Leslie Robb (1988), "Alternative Transformations to Handle Extreme Values of the Dependent Variable", ***Journal of American Statistical Association***, March, Vol. 83, No. 401, pp. 123-27.
10. Christoffersen, P.F. (1998), "Evaluating Interval Forecasts", ***International Economic Review***, 39, pp. 841-62.
11. Christoffersen, P., Jinyong Hahn and Atsushi Inoue (2001), "Testing and Comparing Value-at-Risk Measures", ***Journal of Empirical Finance***, Vol. 8, No. 3, July, pp. 325-42.
12. Dowd, Kevin (1998), ***Beyond Value at Risk – The New Science of Risk Management***, John Wiley & Sons.
13. Engle, R. F. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation", ***Econometrica***, Vol. 50, No. 4, July, pp. 987-1007.
14. Gujarati, Damodar N. (1995) ***Basic Econometrics***, Third Edition, McGraw-Hill International Editions, Economic Series, McGraw-Hill, Inc.
15. Hill, B.M. (1975), "A Simple General Approach to Inference About the Tail of a Distribution", ***Annals of Statistics***, 35, pp. 1163-73.
16. Jarque, C.M. and A.K. Bera (1987), "A Test for Normality of Observations and Regression Residuals", ***International Statistical Review***, Vol. 55, pp. 163-72.
17. John, J.A. and N.R.Draper (1980), "An Alternative Family of Transformations", ***Appl. Statist.***, Vol. 29, pp. 190-97.
18. J.P.Morgan/Reuters (1996), ***RiskMetrics: Technical Document***, Fourth Edition, New York, USA.
19. Jorion, Philippe (2001), ***Value-at-Risk – The New Benchmark for Managing Financial Risk***, *Second Edition*, McGraw Hill.
20. J.P.Morgan Bank (1996), ***RiskMetrics – Technical Document***, New York, J. P. Morgan Bank.
21. Kupiec, P.(1995), "Techniques for Verifying the Accuracy of Risk Measurement Models", ***Journal of Derivatives***, Vol. 2, pp. 73-84.
22. Linden, Mikael (2001), "A Model for Stock Return Distribution", ***International Journal of Finance and Economics***, April, Vol. 6, No. 2, pp. 159-69.
23. Lopez, Jose A. (1998), "Methods for Evaluating Value-at-Risk Estimates", **Research and Market Analysis Group**, Federal Reserve Bank of New York.
24. Mills, Terence C. (1993), ***The Econometric Modelling of Financial Time Series***, 2nd Edition, Cambridge University Press.
25. National Stock Exchange of India Limited (NSEIL)– for data on stock price indices (website www.nse-india.com).
26. Phillips, P.C.B., J.W. McFarland and P.C. McMahon (1996), "Robust Tests of Forward Exchange Market Efficiency with Empirical Evidence from the 1920s", *Journal of Applied Econometrics*, Issue 1 (Jan-Feb), pp.1-22.

27. Reserve Bank of India, Various publications (for exchange rate data). Also see the Bank's website www.rbi.org.in.

28. **Samanta, G.P. (2003), "Measuring Value-at-Risk: A New Approach Based on Transformations to Normality", *Seventh Capital Markets Conference*, December 18-19, Indian Institute of Capital Markets, Vashi, New Mumbai (to apper in the Conference Volume).**

29. Samanta, G. P. and Golaka C. Nath (2003), "Selecting Value-at-Risk Models for Government of India Fixed Income Securities", ***International Conference on Business & Finance***, December 15-16, 2003 at the ICFAI University, Hyderabad, India, Co-Sponsored by the Philadelphia University, USA.

30. Sarma, Mandira, Susan Thomas and Ajay Shah (2003), "Selection of Value-at-Risk Models", *Journal of Forecasting*, 22(4), pp. 337-358.

31. Taylor, Jeremy, M. G. (1985), "Power Transformations to Symmetry", ***Biometrika**,* Vol. 72, No. 1, pp. 145-52.

32. Tsay, Ruey S. (2002), ***Analysis of Financial Time Series***, Wiley Series in Probability and Statistics, John Wiley & Sons, Inc.

33. van den Goorbergh, R.W.J. and P.J.G. Vlaar (1999), "Value-at-Risk Analysis of Stock Returns Historical Simulation, Variance Techniques or Tail Index Estimation?", ***DNB Staff Reports***, No. 40, De Nederlandsche Bank.

34. Wilson, Thomas C. (1998), "Value at Risk", in Alexander, Carol [Ed.], ***Risk Management and Analysis***, Vol. 1, Measuring and Modelling Financial Risk, 1998, John Wiley & Sons.

35. Wong, Michael Chak Sham, Wai Yan Cheng and Clement Yuk Pang Wong (2003), "Market Risk Management of Banks: Implications from the Accuracy of Value-at-Risk Forecasts", ***Journal of Forecasting***, 22, pp. 23-33.

36. Yeo, In-Kwon and Richard A. Johnson (2000), "A New Family of Power Transformations to Improve Normality or Symmetry", ***Biometrika***, Vol. 87, No. 4, pp. 954-59.