#### **Conditional CAPM and Cross sectional Returns - A Study of Indian Securities Market**

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#### **Abstract**

Inability of Fama and Macbeth (1973) two-pass regression procedure to accommodate time varying moments is offered as one of the reasons for the poor cross sectional explanatory power and the anomalies found in Capital Asset Pricing Model (CAPM). We allow the first and second moments of the stock returns to change over time and test three variants of conditional CAPM for value weighted size-based portfolios constructed to test the validity of the same and to capture the 'size effect'. Given the non-normality and temporal dependence in the data, Hansen's Generalized Method of Moments (GMM) has been used for the robust estimation of the parameters. We find that small stock portfolios behave differently from the larger portfolios in terms of their betas and price of covariance risk. While the time variation in beta is significant for the larger stocks, it is not so in the case of smaller stocks. Also the price of covariance risk is found to be inversely related to the size of the portfolios. Given the smaller sample size and highly nonlinear restrictions of the models, size and statistical power of the models are presented.

#### 1 **Introduction**

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Indian stock market has witnessed drastic changes during the past decade under the broad stock market liberalization measures. The screen based trading introduced has made the price discovery process more efficient. Dematerialization of shares and setting up clearing houses has virtually eliminated the risks involved in trading. Similarly rapid strides were made in settlement procedures, corporate governance standards, introduction of derivative products etc. These reforms have increased the participation of Foreign Institutional Investors (FIIs) and other institutional investors in Indian stock market thus widening the investor base and increasing the turn over of the stock exchanges. The impact of all these reform measures reflects clearly on the continuous improvement found in barometers of stock market development such as the number of listed companies, market capitalization, turn over, liquidity etc. Table 1 given below reveals the development of Indian stock market since the 1990s.

Change in the marginal investor with the presence of foreign investors, increased turnover and liquidity, introduction of derivative products and improved regulatory system would all have profound impact on the pricing of financial assets. We undertake to study the process of asset pricing during this period of liberalization under the Conditional Capital Asset Pricing Model (CAPM) framework. Three different models of conditional CAPM have been tested imposing different restrictions to gain important insights on the pricing of financial assets in Indian Stock Market.

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The study is organized in the following way. The first section details the literature on asset pricing especially on CAPM. This is followed by a brief description on the findings of asset pricing studies in Indian stock market. We motivate the selection of empirical specifications/restrictions based on the findings of the previous studies in Indian stock market. The empirical model specification and estimation procedure is described in the third section and would be followed by the data description and important summary statistics. The empirical findings and our interpretation of the results are described in the fifth section. Typically asset-pricing studies of the western literature consider data for three to four decades and our study considers data of only twelve years from 1990 to 2001 on monthly basis. Hence a study on the small sample properties of our empirical specifications is imperative. The size and statistical power of our models are provided in the sixth section followed by concluding remarks in the final section.

## 2 **CAPM and Cross Sectional Returns**

A substantial portion of research in financial economics is devoted to understand how investors evaluate the riskiness of financial assets and the premium attached to the risk. Though it is common knowledge that higher risk is being associated with higher returns, the question that remains are, what type of risks are rewarded and what is the price/reward for bearing the risk i.e. the risk premium per unit of risk. Several equilibrium asset pricing models attempt to answer these questions viz., Capital Asset Pricing Model (CAPM) (Sharpe (1964) and Lintner (1965)) which later got modified by Black (1972), Arbitrage Pricing Theory (APT) (Ross, 1976), Inter-temporal capital

| Year    | No of listed<br>companies | <b>Market</b><br>capitalization<br>(Rs Billion) | <b>MCAP</b><br><b>GDP</b> | Turnover<br>ratio<br>$(\%)$ | <b>Cumulative</b><br>Net FII flows |
|---------|---------------------------|---|---------------------------|-----------------------------|------------------------------------|
| 1990-91 | 6229                      | 1102.79   | 19.39                     |                             |                                    |
| 1991-92 | 6480                      | 3541.06   | 54.21                     |                             |                                    |
| 1992-93 | 6925                      | 2287.80   | 30.57                     |                             |                                    |
| 1993-94 | 7811                      | 4000.77   | 46.56                     | 50.9                        |                                    |
| 1994-95 | 9077                      | 4733.49   | 46.73                     | 34.4                        | 47.96                              |
| 1995-96 | 9100                      | 5722.57   | 48.16                     | 39.7                        | 117.38                             |
| 1996-97 | 9890                      | 4883.32   | 35.69                     | 132.3                       | 203.13                             |
| 1997-98 | 9833                      | 5898.16   | 38.74                     | 154.1                       | 262.70                             |
| 1998-99 | 9877                      | 5740.64   | 32.64                     | 178.3                       | 246.86                             |
| 1999-00 | 9871                      | 11926.30  | 60.94                     | 173.3                       | 348.08                             |
| 2000-01 | 9954                      | 7688.63   |                           | 374.7                       | 447.42                             |
| 2001-02 | 9644                      | 7492.48   |                           | 119.6                       | 534.97                             |

**Table 1: Second market Indicators on Indian Stock Market** 

Source: Indian Securities Market Review - A review, (2002) by National Stock Exchange of India Limited. The GDP figures are obtained from the RBI Handbook of statistics on Indian Economy, 2001. Asset pricing model (ICAPM) of (Merton, 1973) and Consumption based capital asset pricing model (Breedin, 1979). Despite the anomalies found in the CAPM (discussed below), however it still remains the most favorite asset-pricing model for researchers as well as industry practitioners.This can be attributed to its simplicity and intuitive appeal and mainly to the lack of better alternative models<sup>1</sup>.

The CAPM postulates that the return on any asset is linearly related to its market beta, with beta being defined as the ratio of the covariance of the asset with the market portfolio to the variance of the market portfolio. In other words, cross sectionally only the market beta  $(\beta)$  shall be priced. The early empirical tests of CAPM by Black, Jensen and Scholes (1973) and Fama and Macbeth (1973) found support for it because higher returns were associated with higher betas. Although the security market line obtained from their studies were more flat than what is prescribed by CAPM, it was considered to be supporting the zero-beta CAPM of Black (1972).

The problems for CAPM started with the anomalies found in early 80s. The most important of them is 'size effect' (Banz, 1981) i.e. small stocks in terms of market capitalization earn more returns than what is prescribed by CAPM. The 'value effect' (Basu, 1983) says that the high book value / market value (BV/MV) earn higher returns than the low BV/MV stocks. Fama and French (1992) in their widely cited study find that when size and BV/MV factors are considered the CAPM  $β$  has no marginal explanatory power for cross sectional returns. Another important anomaly that cannot be explained by CAPM is the 'momentum effect' (Jagadeesh and Titman, 1993). Stocks that have done well in the past (winners) tend to do well in the future and the losers of the past tend to be lose in the future too and this short term persistence is called momentum. Jagadeesh and Titman attribute this momentum to investors' under reaction.

Fama and French (1993) offers an empirical three-factor model with size and BV/MV considered explicitly apart from the market beta of CAPM.Though the three-factor model have better empirical explanatory power than the original CAPM to explain cross sectional returns the economic reason for why size and BV/MV to be priced is not known. In their later articles, Fama and French (1993,1996) give a reasoning that the small stocks with high BV/MV ratio are firms that have performed poorly and are vulnerable to financial distress and hence command a premium which they call as 'distress premium'. But why distress risk should be priced and why it should command more price than the market beta remain to be answered (Campbell, 2000).

A wide range of explanations are offered to explain away the CAPM anomalies which include data snooping (Lo and Mckinlay, 1990) and behavioral explanations such as investors' over-reaction (Lakhonishok et. al, 1997) or under-reaction (Jagadeesh and Titman, 1993). One of the explanations is that the CAPM might not be able to explain the cross sectional returns unconditionally but conditionally it

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<sup>1</sup> Fama (1991) makes this observation. Also Campbell and Cochrane (2000) explains the poor performance of the consumption based asset pricing models vis a vis CAPM.

might perform well. The assumptions on the estimation of original CAPM, following two pass regression methods of Fama and Macbeth (1973), like constant expected returns; risk premium and market betas are not valid. Risk premiums vary over time (Ferson and Harvey, 1991) and will be higher during the recessionary period. Also during a recession the financial leverage of firms in relative poor shape may increase relative to other firms causing their stock betas to rise (Jagannathan and Wang, 1996). So by allowing the expected returns, betas and risk premiums in the CAPM to vary explicitly, it might be possible to improve the cross-sectional explanatory power of CAPM. Or in other words, the CAPM would perform better conditionally. In this study, we have tested the validity of conditional CAPM for Indian stock market.

#### 2.1 **Asset pricing in Indian context**

Several studies have examined the validity of CAPM for Indian Stock Market for various time periods and come to conflicting results. Detailed description of all the findings are beyond the scope of the article, however important findings that have implications for our empirical specifications is discussed here.<sup>2</sup>

Time variation in beta of the Indian stocks has been reported by various studies. Using Kalman filter and Bayesian structural break model, Verma (1988) finds evidence for time variation in beta of Indian stocks. Moonis and Shah (2002) use a modified kalman filter that can accommodate heteroskedasticity and reject constancy of beta for 26 of the 50 liquid stocks for the time period 1996-2000. Also variation of stock betas for two interest rate regimes (high interest and low interest) is reported by Moonis and Shah (2001). Amanullah and Kamiah (1997) parameterize the second moment (variance and covariance) of the returns as ARCH, ARCH-M, GARCH and GARCH-M processes and find support for conditional CAPM. They also find that GARCH (1,1) performs better than other ARCH and GARCH family processes in explaining the second moment of returns.

To motivate our use of conditional CAPM with time varying risk premium, we present the ex post risk premium on Indian market against the index for industrial production (IIP) in the figure given below.The risk premium is the excess monthly return on BSE-sensex over the short-term risk free rate. The index for industrial production is considered as a proxy for the economic conditions.This would give insights on how the risk premiums changed with changing economic conditions. It can be readily inferred from the figure that the risk premium varies over time and it is negatively correlated (the correlation coefficient is equal to -0.1029) with the index of industrial production implying that the risk premium is more during the recessionary phase than during the expansionary phase. These findings reported justifies the empirical specifications we are using in the later sections of the article.



**Figure 1: Relationship between the risk premium and economic conditions**

#### 3 **Specification of Conditional CAPM**

The conditional CAPM in excess return form can be written as follows:

$$
E[r_{it} / Z_{t-1}] = \frac{Cov[r_{it}, r_{mt}]/z_{t-1}}{Var[r_{mt}]/z_{t-1}} E[r_{mt} / Z_{t-1}]
$$
\n(1)

where  $Z_{t-1}$  is the information available with the investors at time t-1 and is used by them to forecast expected returns, variance and covariance for time t. Hence the expected returns, covariance and variance would be revised at every point of time by the investors based on the new information. In order to test the above model the expected returns, covariance and the variance terms in equation (1) need to be parameterized. For example, Harvey (1989, 1991) use Hansen's generalized method of moments (GMM) and Schwert and Seguin (1990) use the Glejser weighted least-squares estimation approach to find evidence for conditional CAPM. Bollerslev, Engle and Wooldridge (1988), Bodhurtha and Mark (1991) and Ng (1991) employ ARCH/GARCH to parameterize time varying second moments.

Our econometric specifications follow Harvey (1991) to test three variations of conditional CAPM. In all the three forms of conditional CAPM, the return on portfolios  $(r_{i,t})$  as well as the return on market portfolio  $(r_{m,t})$  is defined as

$$
u_{jt} = r_{jt} - Z_{t-1} \delta_j \tag{2}
$$

$$
u_{mt} = r_{mt} - Z_{t-1}\delta_m \tag{3}
$$

 $u_{i,t}$  and  $u_{m,t}$  are the forecast errors on portfolio and market returns, Zt<sub>-1</sub> is the information used by the investors and  $\delta_I$  s their respective coefficients. The assumption here is that the expected returns on the assets and market are linearly related with the information variables.

#### 3.1 **Conditional CAPM with time varying moments**

In this model both the first and second moments of the returns are allowed to vary over time. Applying equations (2) and (3) on equation (1) we would get:

$$
Z_{t-1} \delta_j = \frac{Z_{t-1} \delta_m}{E[u_{mt}^2 / Z_{t-1}]} E[u_{jt} u_{mt} / Z_{t-1}]
$$
\n(4)

By readjusting the terms in equation (3) we will get

$$
E[u_{m}^{2}Z_{t-1}\delta_{j}/Z_{t-1}] = E[u_{jt}u_{mt}Z_{t-1}\delta_{m}/Z_{t-1}]
$$
\n(5)

$$
h_{jt} = u_{mt}^2 Z_{t-1} \delta_j - u_{jt} u_{mt} Z_{t-1} \delta_m \tag{6}
$$

The expected returns on the asset and the market are taken inside the expectation operator because they are known at time t-1. The deviation from the expectation  $(h_{it})$  if divided by the variance of the market return can be considered as the pricing error under this specification. A negative pricing error would imply that the model overprices and positive pricing error would mean underpricing (Harvey, 1991).

The econometric model to tested can be formed by clubbing equations 2, 3 and 5.

$$
\varepsilon_{t} = (u_{t} \quad u_{mt} \quad h_{t}) = \begin{pmatrix} [r_{t} - Z_{t-1}\delta]^{t} \\ [r_{mt} - Z_{t-1}\delta_{m}]^{t} \\ [u^{2}_{mt}Z_{t-1}\delta - \text{Umt}uZ_{t-1}\delta_{m}]^{t} \end{pmatrix}
$$
(7)

This model shall be estimated for every portfolio individually as well as by grouping all the portfolios together. When the model is tested for all the portfolios simultaneously, the test of the overidentifying restrictions do not tell us where the model fails. Similarly, while testing the portfolios individually the results are to be read with caution because one of the important restrictions of CAPM (the price for covariance risk is same for all the portfolios) is not imposed in this case. However this provides test on the restriction that portfolios returns are positively related to its covariance with the market. In order to construct more powerful tests, we have to restrict one of the moments to be constant and test the model.

#### 3.2 **Conditional CAPM with constant betas**

In this version, we restrict the betas of the portfolios (the ratio of the covariance of the portfolio with the market to the variance of the market) to be constant. With this restriction the model reduces to the standard CAPM model with constant beta which is quite common in the literature.

$$
E[r_{it} / Z_{t-1}] = \beta_{im} E[r_{mt} / Z_{t-1}] \tag{8}
$$

 $\beta_{\text{im}}$  is the coefficient of the estimated equation. This model is also similar to single factor latent variables model (Gibbons and Ferson, 1985 and Ferson, 1990) with the only risk premium being the excess return on the market. This model can be tested from the following equation:

$$
k_t = r_t - r_{mt}\beta \tag{9}
$$

where  $k_t$  is the pricing error in this model and should be uncorrelated to the information available at time t-1. No cross restrictions can be imposed in this specification and hence this is estimated for all the portfolios individually.

#### 3.3 **Conditional CAPM with constant price of covariance risk**

In the similar way, conditional CAPM is tested by constraining the price of covariance risk to be constant. The price of covariance risk is defined as the excess return on the market proxy divided by the conditional variance of the market or price offered by the market for taking every unit of market risk. Incorporating the definition for price of covariance risk  $(\lambda_i)$  in equation (1) we get:

$$
E[\text{rit}/Zt-1] = \lambda_i \text{Cov}[r_{it}, r_{mt}]/Z_{t-1}
$$
\n(10)

The pricing error in this version can be written as:

$$
e_t = r_t - \lambda u_t u_{mt} \tag{11}
$$

This model can then be estimated using the following system of equations:

$$
\varepsilon_{t} = (u_{t} \quad u_{mt} \quad e_{t}) = \begin{pmatrix} [r_{t} - Z_{t-1} \delta]^{t} \\ [r_{mt} - Z_{t-1} \delta_{m}]^{t} \\ [r_{t} - \lambda (u_{mt} u_{t}]^{t} \end{pmatrix}
$$
(12)

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This can be simplified because  $E[u_{mt}u_{jt} / Z_{t-1}] = E[u_{mt}t_t / Z_{t-1}]$  and the simplified model can be written as:

$$
\eta_{t} = (u_{mt} \quad e_{t}) = \begin{pmatrix} [\text{Im} t - Z_{t-1} \delta_{m}]^{t} \\ r_{t} - \lambda (\text{Im} t_{t}) ]^{t} \end{pmatrix}
$$
(13)

According to CAPM, the price of covariance risk should be same for all the portfolios.This restriction can be imposed when the model is tested for all the five portfolios together.Besides an additional restriction that the price of risk do not vary over time is also imposed.Hence, rejection of the model in multi portfolio system would either mean that the price of covariance risk is not the same for different portfolios or it is not constant over the time.However if the model were rejected while testing the portfolios individually, it would only mean that the price of covariance risk is time varying.The results of the single asset tests read in conjunction with the result on multiple asset tests would provide all the necessary information regarding  $\lambda_i$ s.

#### 3.4 **Estimation of the Models**

The three version of conditional CAPM models to be estimated are given by equations (7), (9) and (13). These models are estimated using Hansen's (1982) Generalized Method of Moments (GMM). This is because GMM estimates are robust in the presence of non-normality and temporal dependencies in the data. Normality and IID assumptions are generally made in testing asset-pricing models because finite sample properties are derived with these assumptions (Campbell, Lo and Mackinlay, 1997 page no 208). Temporal dependence of the returns or heteroskedasticity in Indian stock market has been reported by various studies including Pradhan and Narasimhan (2002). Similarly the non-normality of the market and portfolio returns is evident from the summary statistics reported in the next section.

The GMM estimation proceeds in the following way. A vector of orthogonality conditions  $g = vec$ (ε'Z) is made where ε is the forecast errors of the model and Z is the array of information variables. The parameters are estimated by minimizing the quadratic form g'wg where w is a symmetric weighting matrix. The consistent estimate of w is given by Hansen (1982) as:

$$
W = \left[\sum_{t=2}^{T} (\varepsilon_t \otimes Z_{t-1})^{'} (\varepsilon_t \otimes Z_{t-2})\right]^{-1}
$$
\n(14)

where ⊗ represents kronecker product. The model can be estimated in two-stage procedure or iterative procedure. In the two-stage procedure, an identity matrix would be used for w to get initial estimate of the parameters and these initial parameters would be used to get the new weighting matrix (w). With the new

weighting matrix, the revised parameters would be estimated. In iterative procedure, the weighting matrix shall be iterated till it converges. We have used iterative procedure for estimating our models.3

The goodness of fit of the model is tested with the minimized value of the quadratic form  $(g<sup>1</sup>wg)$ . Under the null hypothesis that the model is true, the minimized value should be distributed  $\chi^2$  with degrees of freedom equal to the number of orthogonality conditions minus the number of parameters to be estimated. If we assume the number of assets to be n and the number of instrumental variables to be l, then the number of orthogonality conditions and the number of parameters for the conditional CAPM with time varying moments (equation - (7)) should be  $l^{*}(2n+1)$  and  $l^{*}(n+1)$  respectively. Hence the minimized quadratic form should be  $\chi^2$  distributed with degrees of freedom equal to l\*n. The difference between the number of orthogonality conditions and the parameters to be estimated is also known as the number of overidentifying conditions. A higher  $\chi^2$  statistic would mean rejection of the model's restrictions.

#### 4 **Data description and Summary Statistics**

We shall use monthly data from 1990:01 - 2001:12 for 100 stocks listed in Bombay Stock Exchange during the period 1990-2001. These 100 stocks are selected based on the following criteria: (1) The stocks selected should have been listed in Bombay Stock Exchange for the entire period 1990:01 - 2001:12.(2) There should be at least one trading in every month during the time period. (3) The final 100 stocks were selected based on the number of trading days. Five value-weighted portfolios were constructed by value ranking of the companies on the basis of market capitalization at the end of every year and splitting these companies into value-ranked quintiles, and then forming five portfolios based on value weights within a quintile.

The monthly adjusted closing price data for the stocks were collected from the data published by the 'Centre for Monitoring Indian Economy' (CMIE). The call money rate published by the Reserve Bank of India (2001) was used as the short-term risk free rate4. For the market return, we have used the monthly return on the value-weighted index, BSE-National Index of the Bombay Stock Exchange. BSE-National Index comprising 100 stocks is less volatile and broader and hence would serve better as market proxy compared to BSE-30 or NSE-505.

The selection of instrumental variables for conditioning information are is based on western literature (please refer Harvey, 1991 pg no 120 &121 for detailed references and discussion) and the number of instrumental variables is constrained by the number of assets to be tested and the sample size. The sizes of the GMM tests deteriorate rapidly with the increase in the number of instrumental variables (Ferson and Forester(1994), Harvey(1991)). Four instrumental variables that are considered for our study are the lagged market return, foreign exchange rate (Re/USD) changes, difference between the redemption yield on 10 year Government of India bonds and call money rate and finally the ratio of the market proxy (BSE-National index) to the index for industrial production (IIP).

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<sup>&</sup>lt;sup>3</sup> Ferson and Forester (1994) shows that the iterative procedure performs better with small samples.

<sup>&</sup>lt;sup>4</sup> Three-month treasury bill rates are generally used for this purpose. Since it is not available for the whole period call money rates have been used.<br><sup>5</sup> NSE-50 has been back worked till 1990 and is provided by the Nati

The choice of lagged market return is due to high persistence of the market returns in Indian market, which becomes very evident from the summary statistics given in Table 2 below. The difference between the long term interest rate and the short term interest rate gives information on the premium expected for holding the securities for longer time.The exchange rate (Re/USD) movements would directly affect the dollar returns for the foreign institutional investors investing on Indian stocks and hence included as an information variable.The ratio of the BSE-National Index to the index for industrial production gives information on the changing economic conditions and its affect on stock market.The data on the short term interest rate, long term interest rate, monthly average exchange rates and index for industrial production were collected from the Handbook of statistics on Indian Economy - 2001 published by the Reserve Bank of India and International Financial Statistics released by International Monetary Fund.

Table2 gives the basic summary statistics such as mean, standard deviation, skewness and Kurtosis apart from the average market capitalization for the size based portfolios. For the portfolio with the largest stocks (Portfolio I), the average Mcap is 758 billion and the portfolio with smallest stocks (Portfolio V) has an average Mcap of 21 billion. The range of the average market capitalization obtained justifies one of the purpose of this study: to infer size effect. The mean return and the standard deviation are typical of an emerging market return: very high. The mean return is highest for Portfolio 5 (5.55%) and Portfolio 1 has the highest standard deviation of the returns (32.45%) and even the market proxy has a standard deviation of 9.6%.

Another regular feature that is observed on emerging market return data is non-normality.The statistics provided on the skewness and kurtosis justifies that. Besides, we have conducted Kolmogorov-Smirnov test for the normality on the data and the results are reported for 10% significance level in the last column of Table 1. The normality is rejected for Portfolio1, 2 and 4 at conventional levels and for Portfolio 3, 5 and the market return they are rejected at 5% significance level. This justifies our use of Generalized Method of Moments (GMM) procedure to estimate the model.

Table 3 and Table 4 reports the auto correlation properties of the returns, as well as the correlation between the portfolio returns and the instrumental variables respectively. The autocorrelation coefficients and the Ljung-Box Q statistics provide evidence that the Indian stock returns are highly persistent. The correlation between portfolio returns and the market in Table 4 reveals high correlation but counter intuitive. Portfolio 1 has the lower correlation coefficient (0.50) with the market compared to smaller stock portfolios. One would expect larger stock to move more closely with the market. This could be because our Portfolio 1 is more volatile compared to the market proxy, which is a value-weighted index of 100 stocks.

The cross correlation between the portfolios are also very high (Table 3 Panel A). It also reveals another interesting detail. The strength of correlation with other portfolios decreases with the decrease in size.To understand this more we find the cross correlation coefficients with lags because it is quite possible that the larger stocks reflect market information quickly and smaller stocks take more time to reflect market information due to poor liquidity. The results obtained confirm our doubts.

Panel B of Table 4 reports the association between the four instrumental variables that are considered for this study. Since investors use these informational variables to predict the expected returns, the association between these variables are to be lower. Then only the information gained from the four variables would be non-redundant. As expected, the four instrumental variables are not significantly correlated. The highest correlation is between term spread and the ratio market proxy/IIP (0.28).

#### **Table 2: Summary Statistics for the Portfolio Returns**

The statistics are based on monthly data from 1995:02 to 2001:12 (143 observations). The country returns are in excess of the risk free short-term interest rate. Portfolio 1 is the portfolio of largest stocks and Portfolio 5 is the portfolio of smallest stocks. The average value of the Market capitalization is given in Indian Rupee Billion. The test statistic is the Kolmogrov-Smirnov test statistic for normality of the return series for a significance level of 10%.



Note: The test statistic more than 0.10 depicts rejection of normality

## **Table 3: Autocorrelation and Ljung-Box Q statistics**

This table presents the Autocorrelation coefficients at various lags and the Ljung-Box Q statistics are presented in parentheses. The statistics that are significant at 10% level are marked with \*.



**Table 4: Correlation between portfolio returns and Instrumental Variables** 



## **Panel B: Correlation between Instrumental Variables**



## **Table 5: Predictability of Portfolio Returns**

The regressions are based on monthly data from 1990:02 - 2001:12 (143 observations). The portfolio returns are calculated in excess of the short-term risk free rate. T-statistics are given in brackets. The model estimated is:

$$
R_{j,t} = \delta_{j,0} + \delta_{j,1} IV1 + \delta_{j,2} IV2 + \delta_{j,3} IV3 + \delta_{j,4} IV4
$$

The instrumental variables considered are: a constant, the lagged return on the market proxy BSE-National index (IV1), the change in USD/Re exchange rate (IV2), the return on holding 10 year GOI bond minus the call money market rate (IV3) and the ratio of market proxy (BSE-National Index) to the index of industrial production  $(IV4)$ .  $R<sup>2</sup>$  reported is the coefficient of determination adjusted for the degrees of freedom.



The variance of the portfolio returns and the market returns explained by the four instrumental variables are reported in Table 5. Portfolio and market returns are regressed on a constant and the four instrumental variables and the regression coefficients and the associated T statistics are reported. It can be inferred from the table that the coefficients for the exchange rate change in the equation are very low and the T statistics reported are not significant for all the equations. However, other variables significantly affect the returns. As already mentioned, increase in instrumental variables only reduce the size and power of the tests. Hence the exchange rate change is omitted from all the analysis that are reported in the later sections<sup>6</sup>.

## 5 **Empirical Evidence**

For all the models, two statistics are provided for information on the acceptance/rejection of the model restrictions. First, the coefficient of determination reported (R2) for the regression of the model errors on the lagged instrumental variables. For the model's restrictions to be true, the errors should be uncorrleated to the lagged instrumental variables. Higher R<sup>2</sup> would mean that the errors are correlated to the lagged instrumental variables and hence rejection of the null hypothesis. The second statistic is the  $\chi^2$  statistic with the degrees of freedom equal to the number of over identifying restrictions under the null hypothesis. Here also, higher  $\chi^2$  statistic means rejection of the model restrictions.

<sup>6</sup> The inclusion or exclusion of exchange rate as an instrumental variable do not change the results of our model but significantly reduce the size and power of the tests. The results including the exchange rate change are not reported.

Apart from this additional information such as average error and average absolute error are also provided. If the model fits well the error values would be low. Besides inferences such as conditional beta, price of covariance risk is also provided from the models for further inferences.

#### 5.1 **Conditional CAPM with time varying moments**

Table 6 provides the results of the conditional CAPM where the expected returns, variances and covariance are allowed to vary over time. The R2 values reported in Table 6 are very high and imply rejection of the model restrictions.

The  $\chi^2$  statistic is provided for the single portfolio testing as well as the multiple portfolio testing. For the single portfolio testing, the model's restrictions are rejected for all the portfolios except for Portfolio 1. The average error in general is very high and is more than the average returns for all the portfolios. It is positive for all the portfolios implying that the actual returns are higher than the expected returns. The conditional beta obtained from the model reveals that the returns are not related to the conditional beta. This violates the premise of CAPM, the returns are related only to beta and higher the beta the higher the returns. In fact for portfolio 2 and 3, the conditional betas are negative. Surprisingly, the conditional beta is positive and very high for Portfolio 5.

When all the five portfolios are tested together, the model is rejected. By grouping all the portfolios together and testing the model, actually we are imposing the same price of covariance risk for all the portfolios. The rejection of the model can be taken as a rejection for same price of covariance risk.

#### 5.2 **Conditional CAPM with constant beta**

The results for this model are provided in Table 7. The model's restrictions are rejected for Portfolio 1 and Portfolio 2 at conventional significance levels. For smaller stock portfolios, the restrictions could not be rejected. The conditional beta values obtained also seem to have a pattern: the higher the average return the higher the beta except for Portfolio 4. However, the average value of error and the absolute value of error are high for this model also. For example, for Portfolio 5, the value of average error and absolute error is 5.8% and 20.7% for an average realized return of 5.5%.

#### 5.3 **Conditional CAPM with constant price of covariance risk**

Table 8 reports the results for the conditional CAPM with constant price of risk. When tested for the portfolios individually, the model could not be rejected for any of the portfolios. The price of covariance risk  $(\lambda)$  estimated also has a clear pattern. The price of covariance risk is negative for the largest stock portfolio and  $\lambda$  increases clearly with the increase in size. Generally, stocks with higher market capitalization are frequently traded and small size could also represent the poor illiquidity of those

# **Table 6: Estimates of a Conditional CAPM with Time Varying Expected Returns, Conditional Covariances, and Conditional Variances**

Results based on monthly data from 1990:02-2001:12 (143 observations). The portfolio returns 'r' are calculated in excess of the call money rate. The following system of equations are estimated with the Generalized method of moments (GMM):

 $\pmb{\mathsf{I}}$ 

$$
\varepsilon_{t} = (u_{t} \quad u_{mt} \quad h_{t}) = \begin{pmatrix} [r_{t} - Z_{t-1}\delta]^{t} \\ \begin{bmatrix} [r_{mt} - Z_{t-1}\delta_{m}]^{t} \\ [u^{2}_{mt}Z_{t-1}\delta - U_{mt}uZ_{t-1}\delta_{m}]^{t} \end{bmatrix} \end{pmatrix}
$$

where  $r_m$  is the excess return on the world portfolio,  $\delta$  represents the coefficients associated with the instrumental variables, u is the forecast error for the portfolio returns, um is the forecast error for the world market return and h represents the deviation of the portfolio return from the model's expected return. There are three instrumental variables Z that are used in the estimation. They are lagged market return, the difference between the return on holding 10 year GOI bond and call money rate and the ratio of market proxy (BSE-national index) to the index for industrial production.



<sup>a</sup>The average value of  $u_i$  x  $u_m$  returns for country i based on single country estimation divided by the conditional variance of the market.

 $b$ The average value of  $e_i$  for country i based on single country estimation

cThe average absolute value of ei for country i based on single country estimation.

 $d'$ The adjusted coefficient of determination from a regression of the model errors ( $e_{it}$ ) on the instrumental variables.

<sup>e</sup>The minimized value of the GMM criterion function. P-value is the probability that a  $\chi^2$  variate exceeds the sample value of the statistic. For single country system there are 6 parameters and 9 orthogonality conditions leaving 3 overidentifying restrictions. In the multiple equation system there are 18 parameters and 33 orthogonality conditions; this implies that there are 15 overidentifying restrictions to be tested. The degrees of freedom in the test statistic correspond to the number of overidentifying restrictions.

# **Table 7: Estimates of a Conditional CAPM with Time Varying Expected Returns and Constant Conditional Betas**

Results based on monthly data from 1990:02-2001:12 (143 observations). The portfolio returns 'r' are calculated in excess of the call money rate. The following equation is estimated with the Generalized method of moments (GMM):

$$
k_t = r_t - r_{mt} \beta
$$

where  $r_m$  is the excess return on the world market portfolio,  $\beta$  is the proportionality coefficient that relates the expected market excess return to the expected portfolio return, and k represents the deviations from the country returns and the model's expected returns. Three instrumental variables are used in the estimation. They are lagged market return, the difference between the return on holding 10 year GOI bonds and call money market rate and the ratio of the proxy to the market (BSE-National Index) to the index for industrial production.



 $a$  The average value of  $k_I$  for country I based on single country estimation.

 $b$  The average absolute value of  $k_I$  for country I based on single country estimation.

<sup>c</sup> The minimized value of the GMM criterion function. P-value is the probability that  $\chi^2$  variate exceeds the sample value of the statistic. For single country system, there is one parameter and 3 orthogonality conditions leaving two overidentifying restrictions. For multiple equation system there are 4 parameters and 18 orthogonality conditions, which implies that 14 overidentifying restrictions are to be tested. The degrees of freedom in the test statistic correspond to the number of overidentifying conditions.

<sup>d</sup> The adjusted coefficient of determination from a regression of the model errors ( $k_{it}$ ) on the instrumental variables.

# **Table 8: Estimates of a Conditional CAPM with Time Varying Expected Returns and a Constant Price of Covariance Risk**

The results are based on data from 1990:02-2001:12 (143 observations). The portfolio returns r is calculated in excess of the short term risk free rate (call money market rate). Generalized method of moments is used to estimate the system:

$$
\eta_t = \begin{pmatrix} u_{\text{mt}} & e_t \end{pmatrix} = \begin{pmatrix} \begin{bmatrix} \Gamma_{\text{mt}} - Z_{t-1} \delta_m \end{bmatrix}^{\dagger} \\ r_t - \lambda (\begin{bmatrix} u_{\text{mt}} \, r_t \end{bmatrix}^{\dagger} \end{pmatrix}
$$

'

where  $r_m$  is the excess return on the market proxy,  $\delta_m$  values are the coefficients associated with the instrumental variables for estimating the market return, um is the forecast error in the conditional mean of the world return, and λ is the price of covariance risk. In the estimation, the price of risk (expected excess market return divided by the variance of market return) is held constant through time. There are three sets of instrumental variables used in the estimation. They are lagged market return, difference between the return on holding 10 year GOI bond and the ratio of proxy for market (BSE-National Index) to Industrial Production index.



<sup>a</sup>The average value of  $u_i$  x  $u_m$  for country i based on single country estimation.

 $b$ The average value of  $e_i$  for country i based on single country estimation divided by the average conditional variance of the world market return

The average absolute value of  $e_i$  for country i based on single country estimation divided by the average conditional variance of the market return.

 $d$ The adjusted coefficient of determination from a regression of the model errors ( $e_{it}$ ) on the common instrumental variables.

<sup>e</sup>The minimized value of the GMM criterion function. P-value is the probability that a  $\chi^2$  variate exceeds the sample value of the statistic. For single country system there are 4 parameters and 6 orthogonality conditions leaving 2 overidentifying restrictions. In the multiple equation system there are 4 parameters and 18 orthogonality conditions; this implies that there are 14 overidentifying restrictions to be tested. The degrees of freedom in the test statistic correspond to the number of overidentifying restrictions.

stocks. In that case, we can conjecture that the higher price of risk might be to compensate for the poor liquidity.

However, under the framework of CAPM, all these portfolios should have the same price for covariance risk. It is possible to test for this condition by grouping all the portfolios and testing the model. When tested for the group of portfolios, the model is convincingly rejected implying that the price of covariance risk is not the same for different size portfolios.

#### 6 **Size and Power of the GMM Tests**

However, the results reported need to be read with caution for two reasons: sample size and presence of multiple time varying variables. Sample size of Twelve years data on monthly basis or 143 data points is small compared to the western literature where typically three to four years data is used in asset pricing tests. Hence it is imperative to test the size of our statistical tests because our empirical models might over reject or rejects less often for the given sample size7. Similarly, in our conditional CAPM with time varying moments (eqn.7), the expected returns on the market and portfolios, variances/covariance and the price of risk is allowed to vary over time. In the event of model being rejected, which is incidentally the case in our study, we do not know where the model fails.However, tests of two other variants of conditional CAPM models presented in Eqn (9) and (13) can be considered statistically more powerful because it restricts one of the variables to be constant. Hence this section is devoted in studying the size and power of our tests.

The conditional CAPM models we have used have highly non-linear and cross-equation restrictions. We follow Ferson and Forester (1994) and use simulation procedure to estimate the size and power of the models tested. They provide a rationale that the GMM is likely to be sensitive to moments in the data not matched by the artificial economies. Hence the artificial data is generated by resampling the data similar to bootstrapping procedure. By adopting this procedure the essential statistical properties of

 $\overline{a}$ 

<sup>7</sup> We thank the anonymous referee for this useful suggestion.

the data is retained. To generate artificial data that satisfy particular conditions, appropriate moment restrictions are imposed.

The procedure we have adapted to generated data for the artificial economies under various conditions are described in detail in the Appendix. Before going to the size and power of the tests, we have tested the accuracy of the bootstrapping procedure by checking whether it provides reliable finite sample distributions. For all the three variants of the conditional CAPM model we have used, we generate 1000 samples of artificial data. The average fraction exceeding the critical values for  $\chi^2$  distribution with appropriate degrees of freedom is documented. This gives the idealized true sampling distribution. Five of the idealized data samples are picked at random and their distribution is reported. For the boot strapping procedure to be accurate, the distribution of the samples picked should not vary from the idealized true sampling. Table 9a gives the results. It is evident from the table that the distributions of the five random samples do not vary much from the idealized true sampling. Hence we proceed to test the size and power of the tests.

## 6.1 **Size of the GMM Tests**

From the Table 9a we can infer the size of the tests. The fraction of the idealized true sampling distribution exceeding the critical values at various significance levels are reported for the three variants of the conditional CAPM models. In general, the conditional CAPM with time varying moments (for the single asset system) is rejected too often. Even in the conventional significance levels such P=0.01, 0.05 and 0.10 the rejections are 0.06, 0.24, 0.35. This means the model over rejects six times, five times and three times for P=0.01,0.05 and 0.10 respectively. The rejection of this model for Portfolio 2,3,4 and 5 can be attributed to this over rejection.

For the conditional CAPM with constant beta, the results are much better. The rejections for P=0.01, 0.05 and 0.10 are 0.01, 0.08 and 0.16 respectively. Over all the performance of this model is satisfactory and superior to other models. The situation is different for the conditional CAPM with constant price of risk: The model is slightly under rejected at conventional significance levels. This could also be a reason why the model is accepted for all the portfolios.

#### 6.2 **Statistical power of the Tests**

Having tested the size of our tests, we go ahead by testing the power of our tests. The statistical power of the test is the probability that the null hypothesis will be rejected given that an alternative hypothesis is true. For example, to test the statistical power of the conditional CAPM with constant beta (Eqn (9)) against an alternative hypothesis of time varying beta, we generate artificial data for time varying beta (as described in the appendix) and test whether the constant beta model is rejected.

The results of the power tests for all the three variants of our conditional CAPM for various alternative hypotheses are reported in Table 9b. The power is computed for tests with size of  $\alpha = 0.01$ , 0.05, 0.10, 0.25 and 0.50. As reported before, some of our tests reject too often and some under rejects and hence the critical value of the test statistic needs to be adjusted for the same. We get an adjusted critical value, the value that is exceeded by  $\alpha$  fraction of the statistics under each null hypothesis. Now the statistical power of our tests is the fraction of 1000 trials in which a test using the adjusted critical value rejects the null hypothesis when the data is generated by a different specification.

The power of our conditional CAPM model with time varying moments (Eqn 7) is reported against two alternative conditions: when the beta is constant and when the price of covariance risk is constant. For both the cases, the power of the model to reject the null hypothesis is very low. For the constant beta model (Eqn 9) the statistical power against the time varying betas and the constant price of covariance risk alternative is good. The conditional CAPM with constant price of covariance risk has good power against the time varying price of risk as well as the constant beta model. Overall, our conditional CAPM with time varying moments have very low power against alternatives. However, conditional CAPM with constant beta and conditional CAPM with constant price of covariance risk are powerful to identify the data generated from alternative hypotheses.

On the basis of the size and power tests, we can infer that the results of the conditional CAPM with time varying moments (Eqn 7) should not be taken too seriously. This is

|  | <b>Fraction Exceeding critical values</b> |       |       |       |       |       |  |  |  |  |  |
|--|---|-------|-------|-------|-------|-------|--|--|--|--|--|
|  | 0.50                                      | 0.25  | 0.10  | 0.05  | 0.025 | 0.010 |  |  |  |  |  |
| Conditional CAPM with Time varying Moments (N=1, L=3, T=143)   |   |       |       |       |       |       |  |  |  |  |  |
| <b>Idealized</b><br>true<br>sample                             | 0.782                                     | 0.576 | 0.363 | 0.229 | 0.136 | 0.072 |  |  |  |  |  |
| Experiment 1   | 0.789                                     | 0.585 | 0.355 | 0.242 | 0.134 | 0.061 |  |  |  |  |  |
| Experiment 2   | 0.784                                     | 0.575 | 0.357 | 0.234 | 0.142 | 0.077 |  |  |  |  |  |
| Experiment 3   | 0.777                                     | 0.538 | 0.332 | 0.220 | 0.132 | 0.066 |  |  |  |  |  |
| Experiment 4   | 0.773                                     | 0.561 | 0.334 | 0.226 | 0.135 | 0.066 |  |  |  |  |  |
| Experiment 5   | 0.776                                     | 0.573 | 0.343 | 0.226 | 0.155 | 0.073 |  |  |  |  |  |
| <b>Conditional CAPM</b> with constant beta                     |   |       |       |       |       |       |  |  |  |  |  |
| Idealized<br>true<br>sample                                    | 0.618                                     | 0.358 | 0.164 | 0.083 | 0.045 | 0.020 |  |  |  |  |  |
| Experiment 1   | 0.629                                     | 0.372 | 0.160 | 0.081 | 0.041 | 0.016 |  |  |  |  |  |
| Experiment 2   | 0.637                                     | 0.384 | 0.166 | 0.087 | 0.046 | 0.021 |  |  |  |  |  |
| Experiment 3   | 0.618                                     | 0.359 | 0.168 | 0.085 | 0.047 | 0.015 |  |  |  |  |  |
| Experiment 4   | 0.635                                     | 0.388 | 0.163 | 0.085 | 0.041 | 0.017 |  |  |  |  |  |
| Experiment 5   | 0.622                                     | 0.351 | 0.154 | 0.084 | 0.044 | 0.016 |  |  |  |  |  |
| <b>Conditional CAPM with Constant Price of covariance risk</b> |   |       |       |       |       |       |  |  |  |  |  |

**Table 9: Accuracy of boot strapping procedure and size of the tests** 



because the model is rejected too often and it has very low power against alternative hypotheses. On the contrary, the other two models (Eqn 9 and 13) perform well even for the smaller sample size. There is a clear size effect on Indian stocks. The betas of the larger stocks (Portfolio 1 and Portfolio 2) are time varying but the betas of small stocks do not vary over time. The price of covariance risk is not the same for all the portfolios and it increases with the reduction in size of the portfolios. Higher price of risk for small stock portfolios could be because of its relatively poor liquidity and this finding need to be studied further.

## **Table 9A: Adjusted critical values for various null hypotheses**





## **Table 9B: Statistical power of the tests**

## 7 **Conclusion**

We have tested three variants of the conditional CAPM for value weighted size based portfolios for Indian stock market. The conditional CAPM with time varying moments is rejected for all the portfolios except for the portfolio of largest stocks. This has to be read with caution because this model is rejected too often given the sample size and this test lacks power to differentiate alternative hypotheses. The other two models, conditional CAPM with constant beta and conditional CAPM with constant price of covariance risk perform better for small samples and possess statistical power too. Based on this, we were able to infer that the betas of the portfolio comprising large stocks vary over time and the small stock portfolio betas do not vary. Also the price of covariance risk is not the same for all the portfolios and is inversely related to the size.

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## **Appendix**

The Procedure for generating artificial data for various restrictions is explained below.

## **Conditional CAPM with constant beta**

- 1.  $r_{mt}$  is regressed on the three instrumental variables ( $Z_{t-1}$ ) and the coefficients ( $\delta_m$ ) is obtained.  $\delta_m Z_{t-1}$ gives the expected market return.
- 2. The portfolio returns are regressed on the expected market return  $\delta_{m}Z_{t-1}$  and the coefficient obtained  $(\beta_{im})$  is used as the true beta coefficients for the portfolios.
- 3. The expected returns for the portfolios are obtained as the product of  $\beta_{\text{im}}$  and the expected market return  $\delta_{m}Z_{t-1}$ .
- 4. The forecast errors on the portfolio returns can be obtained using the formula:

 $e_{it} = r_{it} - \beta_{im} (\delta_m Z_{t-1})$  for  $i = 1, 2...5$ 

- 5. To generate an observation of artificial returns for each data t ( $t=1,2,...$ . T), one random integer  $t^*$  is drawn,  $1 \le t^* \ge T$  and the error value at time t from the error series created in the previous step  $e_t^*$  is selected as the error for time t.
- 6. The artificial returns are generated using the following formula:

 $r_{it} = \beta_{im} (\delta_m Z_{t-1}) + e_t^*$ 

- 7. With the artificial returns generated, the model is estimated using the GMM procedure. Since the iterative GMM procedure is used to test the original data, the same procedure is used for the artificial returns.
- 8. The steps 5-7 is repeated for a total of 1000 replications.

#### **Conditional CAPM with constant price of risk**

- 1.  $r_{mt}$  is regressed on the three instrumental variables  $Z_{t-1}$  and the coefficients  $(\delta_m)$  is obtained.  $\delta_m Z_{t-1}$ gives the expected market return and  $(r_{mt} - \delta_m Z_{t-1})$  gives the value of u<sub>mt</sub>.
- 2.  $r_{it}$  is regressed on a constant and the product of the forecast error on the market return and the portfolio return ( $u_{m}$  $_{t}$ ;). The slope coefficient obtained is the price of covariance risk ( $\lambda$ ) is retained as the true value of the price of covariance risk to generate artificial data.
- 3. The expected returns (artificial) for the portfolios are obtained as  $(\lambda u_{m}t_{it})$  and the forecast error of portfolios returns ( $e_{it}$ ) would be  $r_{it}$  - ( $\lambda u_{mt}r_{it}$ ).
- 4. Similar to the step 5 described above, one error value  $(e_{it}^*)$  shall be picked for each time t and the error series shall be generated for the whole time period.
- 5. The artificial returns would be  $(r_{it}^1) = (\lambda u_{mt} r_{it}) + (e_{it}^*)$  for every t,  $1 \le t \ge T$ .
- 6. Step 7 and 8 described are repeated.

#### **Conditional CAPM with time varying moments**

- 1.  $r_{\text{mt}}$  is regressed on the three instrumental variables  $Z_{t-1}$  and the coefficients ( $\delta_{\text{m}}$ ) is obtained.  $\delta_{\text{m}}Z_{t-1}$ gives the expected market return and  $(r_{mt} - \delta_m Z_{t-1})$  gives the value of u<sub>mt</sub>. Similarly  $r_{it}$  is regressed on the instrumental variables  $Z_{t-1}$  to obtain the expected return on the portfolios ( $\delta_i Z_{t-1}$ ) and the forecast error u<sub>it</sub>.
- 2. For the single asset system the time varying beta  $(\beta_i)$  can be obtained by dividing the covariance of the portfolio return with the market return ( $u_{it}u_{mt}$ ) by the variance of the market return ( $u_{mt}$ )<sup>2</sup>. This shall

be used as the true time varying beta for the portfolio to generate artificial data. However in the case of multiple asset system, the restriction that is imposed is that the price of covariance risk is time varying but common for all the portfolios. For that purpose, the true price of covariance risk is obtained by taking the ratio of expected market return ( $\delta_{m}Z_{t-1}$ ) to the variance of the market return  $(u_{\text{mt}})^2$ . The artificial returns for the portfolios shall be obtained using the price of covariance risk.

- 3. The pricing error of the model (h<sub>it</sub>), is obtained by taking the difference between ( $\delta$ i Z<sub>t-1</sub>) and  $\beta$ i (  $\delta_{m}Z_{t-1}$ ).
- 4. The pricing error ( $h_{it}$ ) and the forecast error for the portfolio returns ( $e_{it}$ ) is combined to form the total error (te<sub>it</sub>).
- 5. One value shall be picked on random from the total error series  $(te<sub>i</sub><sup>*</sup>)$  to obtain the total error for every time t,  $1 \le t \ge T$ .
- 6. The artificial returns are generated from the following formula:

$$
r_{it}^{1} = (\delta_{i} Z_{t-1}) + \beta_{i} (\delta_{m} Z_{t-1}) + t e_{it}^{*}
$$

7. The artificial returns are estimated using GMM and this procedure shall be repeated 1000 times.

## **Time varying betas and time varying price of covariance risk**

- 1. The expected market returns as a linear function of the instrumental variables are obtained as in other procedures.
- 2. For time varying beta economy, the conditional betas are assumed as the linear functions of the instrumental variables say

 $\beta_{im} = \gamma Z_{t-1} = constant + \gamma_1 Z_{1,t-1} + \gamma_2 Z_{2,t-1} + \gamma_3 Z_{3,t-1}$ 

3. The coefficients γ is obtained by regressing the asset return on the market index return

 $r_{i, t}$  = (constant +  $\gamma_1 Z_{1, t-1}$  +  $\gamma_2 Z_{2, t-1}$  +  $\gamma_3 Z_{3, t-1}$ )  $r_{mt}$ 

4. From the coefficients  $\gamma$  and the instrumental variables, the time varying beta is obtained.

In case of time varying price of covariance risk, the  $\lambda$  is expressed as a linear function of the instrumental variables and the rest of the procedure is similar.