Econometric Estimation of Systematic Risk of Nifty-Fifty Constituents of the Indian Stock Market

January 17, 2001

1.1 Introduction

This study deals with assessment of systematic risk of equity stocks, which is an important issue in the modern theory of finance and has received the attention of a number of financial theorists over the past three and a half decades. Empirical research on this issue has, in general, been carried out in various countries, where daily quotes of stock prices are available electronically in the form of comprehensive data files. Estimation problems particularly arising from infrequent trading of stocks could be studied in detail, only with the availability of such comprehensive data inclusive of those infrequently traded stocks. With the advent of internet technology in India, such authentic data files have recently become available to researchers, which has facilitated the present study.

1.2 Nature of the Problem

In financial econometrics, systematic risk of investing in any stock is generally represented by the slope coefficient, $\beta$ in the market model

$$R_t = \alpha + \beta M_t + e_t \ldots \ldots (1.1)$$

where, $R_t$ and $M_t$ represent true returns on an equity stock and on the market portfolio (or a stock market index) respectively; $\alpha$, the intercept term and $e_t$, the random error at time $t$. Given a sample of observations of prices and traded values of any market index, the returns-variables $R_t$ and $M_t$ are generated from them. Using these values, unbiased estimates of $\beta$’s can be obtained by the classical method of Ordinary Least Squares(OLS), provided the error structure satisfies the standard OLS assumptions.

However, when some of the stocks comprising the stock market index are intermittently traded, the returns on the market index $M_t$ would be serially dependent. This problem as pointed out by Fishier(1966) would render the estimates of $\beta$ biased. The bias is caused by serial correlation of residual errors in regression of the market model above. A positive serial correlation is induced into $M_t$ and its estimated covariance of $R_t$ and $M_t$ will be biased downward for infrequently traded stocks. Since, by definition, the value of $\beta$ is unity for the market portfolio, estimates of $\beta$ will be biased upward for more frequently traded stocks.

1.3 Infrequent (non-synchronous) Trading and Consequences
Infrequent (or non-synchronous) trading is commonly associated with small companies quoting very low share price, or for even large companies traded in emerging stock markets which are at their early stages of development and which are undergoing continuous regulatory reforms such as in India. The returns generating process is influenced by a tendency in prices ($p_t$) at the end of a time period, $t$, to represent the outcome of a transaction, which occurred prior to the period in question. An important consequence is that it makes a scrip illiquid, and, therefore generally discourages investors for further trading in the same. Sometimes, high value of a share price discourages small investors to participate in trading, making the scrip relatively less traded than otherwise, despite good financial results in the past and/or bright prospects for a repeat or better performance in the future. Many important large companies in India (like Infosys Technologies, Satyam Computers, Hindustan Lever etc.,) having high market capitalization have recently resorted to stock splits, perhaps, to improve liquidity and trading opportunities.¹

In case daily close prices are considered for estimation of some stocks, which are not necessarily traded every day, observations assume no value for any non-trading day. This would mean that the corresponding return variable $R_t$ has ‘no specific value (not zero value) for that day. If, however, close prices for a non-traded stock are reported to be the same as for the previous trading day, the corresponding return variable could be incorrectly assumed to have a value zero; whereas, $M_t$ will represent the returns on the index which will be a weighted average of temporally ordered values of other traded index stocks. Sometimes, close prices are quoted for some stocks, which are traded once or twice in a day, but not necessarily at the end of the trading hour. When daily closing prices are used for estimation, it would be tantamount to assuming that their traded prices are equally spaced at a 24-hour interval, which is, strictly speaking incorrect. Serious auto-correlations are possible in daily prices whenever a company announced corporate actions like bonus, stock splits after the closing hours of a trading day, because the outcome of that day’s action will influence the opening (as also the closing) price the next day with a lead time. In the case of daily quoted stocks, it is also known that intra-day trading is more frequent for some leading stocks than for other stocks included in the market index. Non-synchronous trading thus imparts serious bias in the mean, variance, and correlations of asset returns (see Campbell, Lo and MacMackinlay(1997, p.84).

Trading frequency of a stock also depends on the time interval during which quotes are observed. It is likely that, higher will be the trading frequency, the longer the time interval. When a stock is intermittently traded, its effect can thus be observed significantly with daily quotes, rather than with the corresponding monthly or annual returns computed thereof using the daily prices. Estimation of risk from a market model using monthly or annual returns could subsume the bias due to infrequent trading. This is referred to as the intervaling effect in the literature. The intervaling effect is noted to be a consequence of infrequent trading problem (Schwartz and Whitcomb, 1977). Also, infrequent trading could be because of a wide difference between sell-price and bid-price, popularly known as “bid-ask price spread”.

It is also a common practice to estimate a market model by considering data for longer intervals of time, say using monthly, quarterly or annual periods, which are in turn generated from averages of daily observations, referred to as ‘data massaging”. This could circumvent the problem of infrequent trading. Even then, OLS estimates of market model parameters will be biased due to auto-correlation. There will, however, be a tendency for such regression results to yield higher values of the coefficient of determination and betas.

1.4 Organization of the study

¹ Though Satyam Computer Services and Infosys Technologies are highly liquid stocks, their trading frequency can be influenced adversely when their share prices are relatively high, by Indian standard, despite the fact that investors can buy even one single share under the De-mat facility.
Dimson(1979) provides a theoretical framework to estimate unbiased estimates of $\beta$’s under such a situation, which is adopted in this paper to derive the estimates of systematic risk of fifty different stocks that comprise S&P-CN X NIFTY (in short, NIFTY-50) market index of the Indian stock market. The market risk is assessed for each of these fifty stocks using daily data of their closing prices, and the daily closing values of NIFTY-50 market index for the period beginning November 3, 1995 to May 23, 2000. This paper is organized as follows. Section 2 deals with the methodological steps involved in estimation, while section 3 provides details of variables and original data used and subsequently, various price adjustments carried out to incorporate corporate actions such as announcements of dividends, rights, bonuses and stock splits that theoretically affect the prices observed during the period of study. Section 4 presents the findings and results of this empirical exercise and offers a comparison with betas estimated by classical simple model using the same data set, and tests the significance of bias resulting therefrom. Finally, section 5 gives a summary of findings and offers limitations of using these estimates for the purpose of investment and policy decisions.

2. Methodology

2.1 Three approaches to estimation

Prior to the Dimson(1979) study, there were attempts by other researchers in the literature to assess market risk of less frequently traded stocks, which can be categorized into three approaches. The first approach takes into account the lagged market returns as additional independent variables in the market model (1.1). In the second approach, whenever there is a gap in trading period, returns are computed for infrequently traded stocks on a trade-to-trade basis and, in the same manner, returns of the market index are also computed. Then the market model is estimated using the OLS method. In the third approach, Scholes and Williams(1977) combined the two and used the current and lagged market returns as explanatory variables to estimate the market model. Dimson’s method is a generalization of these three approaches, which provides a general framework to address the question why the classical market model yields biased estimates of betas, when some stocks are infrequently traded, and attempts to remedy the problem.

2.2 Dimson’s Aggregate Coefficients (AC) Method

According to the Dimson’s AC method, when a stock is infrequently traded, the market model is estimated with some modifications; that is, by incorporating in the regression the lagged and leading market returns variables (non-synchronous) in addition to the current market returns (synchronous) as the independent variables. The Dimson model is given as follows.

$$\hat{R}_t = \alpha + \sum_{k=-n}^{n} \hat{\beta}_k \hat{M}_{t+k} + \hat{w}_t \ldots \ldots (2.1)$$

where $\hat{R}_t$ and $\hat{M}_t$ are the observed returns on the stock and the observed returns on the market index in period $t$ (current) respectively, and $\hat{w}_t$ the error term with zero mean, zero covariance with $\hat{M}_t$ and has a constant variance.

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3 Marsh(1979), Schwert(1977) and Franks et al., 1977 used this approach.

4 See in particular section 2.2 of Dimson(1979, p.202) for this framework.
Since securities are traded intermittently, an observed price \( p_t \) may represent a transaction price \( p_{t-i} \) in the current period \( t \), or, a transaction price occurred in an earlier period \( p_{t-i} \) for \( i > 0 \).

Observed prices are used to generate the observed returns on a stock as well as on the market portfolio. Since observed prices have a chance of occurrence, they have an expected value which is the weighted average of a sequence of true transaction prices. That is,

\[
\hat{E}(p_t) = \sum_{i=0}^{n} \theta_i p_{t-i}
\]

which can be expressed in terms of observed returns on a stock as follows.

\[
\hat{E}(R_t) = \sum_{i=0}^{n} \theta_i R_{t-i}
\]

where \( \theta_i \) represents the probability of a security having been traded in the period \( (t-i) \) for \( i \geq 0 \). For instance, for \( i=0 \), \( \theta_0 \) gives the probability that it is traded in the current period (trading frequency of a stock in the current period). It is also assumed that \( \theta \)'s are stationary and identically distributed over time, and further, that

\[
\theta_i \geq \theta_{i+j} \text{ for } j > 0, \quad \sum_{i=0}^{n} \theta_i = 1.
\]

Analogous expression for the market returns is given by

\[
\hat{E}(M_t) = \sum_{i=0}^{n} \phi_i M_{1-i}
\]

where \( \phi_i \) represents the probability of the market index having been traded in the period \( (t-i) \) for \( i \geq 0 \). To interpret, for instance, for \( i=0 \), \( \phi_0 \) gives the probability that it is traded in the current period (trading frequency of the market index in the current period). It is also assumed that \( \phi \)'s are stationary and identically distributed over time, and further, that

\[
\phi_i \geq \phi_{i+j} \text{ for } j > 0, \quad \sum_{i=0}^{n} \phi_i = 1.
\]

It should be noted that \( \text{Cov}(R_t, M_t) \) varies directly with trading frequency. For infrequently traded stocks, the values of \( \theta_i \) and \( \phi_i \) are lower and hence the covariance between the observed stock returns and the observed market returns will be lower than that for frequently traded stocks.

Since \( \beta = \frac{\hat{\text{Cov}}(R_t, M_t)}{\sigma^2_m} \), where \( \sigma^2_m \) is the variance of the returns on the market portfolio, and since the mean value of \( \beta \) for the market portfolio is unity, it follows that the bias in estimation of \( \beta \) is downward for infrequently traded stocks and upward for more frequently traded stocks. When all stocks included in a market index (which is value weighted) are estimated by a simple regression of \( R_t \) on \( M_t \), the distribution of betas so obtained will be affected according as their bias. The cross-sectional variance of biased beta estimates will also be adversely influenced.
Using the relationships between true and observed returns as given above, Dimson showed that an unbiased estimate of systematic risk in equation 1.1 can be obtained from:

\[ \hat{\beta} = \sum_{k=-n}^{n} \hat{\beta}_k \]  

(2.2)

where it is assumed that securities are traded at least once in every ‘n’ periods. This implies that, when some of the constituent securities of a market index are infrequently traded, an unbiased estimate of systematic risk of a security is obtained by first estimating the associated regression coefficients(\(\beta_k\)) of synchronous and non-synchronous(the lagged and the leading) market returns variables in the modified market model at equation 2.1, and then by summing them up as shown in 2.2. Dimson notes that for infrequently traded stocks, the estimated values of lagged coefficients, (\(k<0\)) are negatively related to the trading frequency, while for frequently traded stocks, the estimated values of leading coefficients, (\(k>0\)) are positively related to the trading frequency. If the trading frequency of a stock(\(\theta_0\)) in the current period is relatively less than that of the market index (i.e., a stock is infrequently traded as compared to the market), then it is the lagged coefficients which are more important, while for very frequently traded stocks it is the leading coefficients which are very important. So much depends on how the market index is composed of.

For conducting empirical estimation, the choice of ‘n’ is important. Since ‘n’ indicates the period during which any stock included in a market index is traded at least once, any assumption about it must take into account the knowledge of comprehensive data base of share prices of all securities comprising the market index. For instance, if the available database suggests that every stock of the index is traded at least once in a 5-day trading week, then ‘n’ assumes a value 5. Or, if the data base shows that there are some stocks of the market index which are traded once in a while during the data period of study, but its trading frequency is such that there are not more than two trading gaps consecutively, then ‘n’ assumes a value 1. Furthermore, the choice of how many lags and leads to be incorporated into the model (the values of k and n) is also dependent upon the cross-sectional mean and variance of coefficients, which are estimated with some pre-supposed values of k and n. Dimson has provided a working formula for a final choice of ‘n’ as follows.

Suppose \(\beta_k\) denotes the true value of the coefficient and \(\hat{\beta}_k\) its estimated value with some pre-supposed value of ‘n’.

Let \(\text{var}(\hat{\beta}_k)\) denote the corresponding sample variance. Then, the variance of true value of \(\beta_k\) (written as \(\text{VAR}(\beta_k)\)) is estimated as the difference between the cross-sectional variance of \(\hat{\beta}_k\) (\(\text{VAR}(\beta_k)\)) and the cross-sectional mean of sample variances. That is

\[ \text{VAR}(\beta_k) \equiv \text{VAR}(\beta_k) - \text{MEAN}(\text{var}(\beta_k)) \]  

(2.3)

\footnote{For instance, suppose a security is traded any time during the period of study such that it has quoted a price on the first, fourth and fifth trading days, then there is a gap on the second and the third days. In that case, ‘n’ should assume a value 2.}
Thus, in order to estimate the variance of true value of $\beta_k$ of a stock, one needs to consider a cross-sectional sample of stocks that comprise a market index, and by assuming some value of 'n' on the basis comprehensive data base information, obtain the OLS estimates of $\beta_k$'s of all stocks, and their respective sample variances. Using these, one must get the cross-sectional mean of these sample variances, and the cross-sectional variance of the estimated $\beta_k$'s, where k varies from $-n$ to $+n$. Dimson observed that the contribution of the lagged and leading market returns variables to produce an unbiased estimate of beta will be optimum when the above expression (2.3) attains a small value and tends to fluctuate around zero, as 'n' is increased. It thereby gives a cue for the appropriate choice of 'n'.

3. Data Arrangement, Features and Adjustments of Returns

3.1 Data arrangement

The daily data of close prices of all the fifty stocks comprising the market index, namely, S&P-CNX NIFTY, as well as the daily traded values of the market index listed on the National Stock Exchange (NSE) have been made available electronically by the Mumbai office of NSE for the period from January 1, 1995 to May 23, 2000. The stocks, which appeared in the market index as of April 18, 2000, have been considered in this empirical exercise. Since the market index itself was constructed since November 3, 1995 (one year later than the date of commencement of on-line trading in India), the study period has been fixed from November 3, 1995 to May 23, 2000.

The history of trading frequency was examined in detail during the study period for the 50 stocks comprising the market index as of April 18, 2000. The degree of non-trading, (or, infrequent trading) has been identified with as many as 10 out of 50 stocks and the details of non-traded days as well as the number of price observations on each of them, and data period of different stocks are reported in Table-1. It shows that the sample size (number of observations) for very frequently traded stocks is 1135.

3.2 Non-synchronous trading in the Sample

The cross-section of companies considered in the sample suffers from two important features of non-synchronous trading. One, as far as infrequently traded stocks in the sample are concerned, there are only 10 stocks with history of infrequent trading in the sample. Britannia Industries did not trade for a maximum number of 27 days, followed by Procter and Gamble(P&G) which was not traded for 22 days, Hindustan Petroleum (8 days), Infosys Technologies(7 days), BHEL and Zee Telefilms(3 days each), and four other stocks not having been traded for less than 3 days during the time period of study. Bank of India has a shortest trading history in the whole cross-section, as it gained listing only in August, 1997. But it was traded on all days since it was listed. Accordingly, the number of observations available for estimation varied from 761 to as high as 1136. It implies that the sample size for regression of the market model of stock

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6 Although it gives some indication to get at a right choice for 'n', it is not clear whether or not one must take equal number of lags and leads (i.e., should 'k' always run from $-n$ to $+n$?). Introduction of too many lags and leads into the market model might also result in estimation error of $\beta_k$'s. He had also referred to alternative ways whereby the beta estimate may be explicitly adjusted back to their mean value to take account of the estimation error of $\beta_k$'s.

7 The observations for July 22, 1996 appear twice as the second one refers to data of special trading session on the budget day. Similarly, the sample size includes an observation related to the budget session on February 28, 1997.
returns (without any lags or leads of market returns) has varied between 760 to 1135, since one observation is lost in generating the returns variable.

The second feature of non-trading is more important for the choice of ‘n’. It is about the string of consecutive days on which any stock in the sample did not trade, which, according to Dimson methodology, indicates the value of ‘n’ one can presuppose in order to apply a desirable number of leads and lags in the estimation of betas. On a careful examination of trading frequency of individual stocks (see Table-1), it is found that consecutive non-trading is not more than a day, and therefore, ‘n’ can be prescribed a value 1. That is, the model should contain at least one lead and one lag in order that the Dimson’s AC method yields an unbiased estimate of beta for each of the 50 stocks. Of course, a higher number of leads and lags have also been tried in this exercise as a check to identify the right number for ‘n’. At the other extreme, it is assumed that each stock in the cross-section has traded at least once in a 5-day trading week. In summary, Dimson’s AC method has been used to obtain unbiased estimates of betas, under the option of a) one lead and one lag and b) 5 leads and 5 lags, and the results of estimated coefficients, $\beta_k$’s are employed to conclude in which case the true beta of a stock has the lowest variance as per the formula (2.3) given earlier.

3.3 Price Adjustments

Before generating stock returns from daily data of close prices, it is necessary to incorporate in prices dividend payments and capital changes due to corporate actions like payment of bonuses, issue of rights, and stock splits by the companies during the time-period of study, as such changes would affect the implicit returns on stocks held by investors. Dividend payments are made to the shareholders whose names appear in the books of accounts as on the record date, announced by the company concerned and notified to the stock exchange. For the purpose of estimation, dividend payment per share is added to the ex-dividend price (i.e., the price prevailing on the day when no-delivery period begins), and the daily return relevant for that date is computed. Similarly, price change due to bonus payment is carried out on the basis of bonus ratio as on the ex-bonus date (i.e., when the no-delivery period for bonus payment begins) on the stock exchange, and the stock return on the ex-bonus date is derived by the actual price prevailing on that date minus the theoretical price, worked out on the basis of bonus ratio.

Adjustments due to stock splits are also made in the same manner as in the bonus case. When a stock quoting at Rs.2100 per share is split in the ratio of 1 into 5, (as it happened in the case of Satyam Computers recently), its theoretical ex-splits price is 420. If this share is traded at any price above this level, say, 450 on the first day of the no-delivery period for the purpose of recording stock splits in the books of accounts of the company, it is construed to earn a capital gain of about 7.14%, or else, if it is quoted below 420, a capital loss.

In the case of rights issue, the theoretical ex-rights price and stock returns thereof have been computed on the basis of weighted average of the cum-rights price and the offer price, where weights are taken using the ratio in which rights are offered to

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8 For instance, if a share quoting a price of Rs.2200 earns a bonus in the ratio of 1:3 (one for three held), then its theoretical price ex-bonus will be Rs.2200x3/4 = 1650. If the actual price on the first day of no-delivery period (the ex-bonus date) is quoted at 1800, then it yields a stock return of 9.09%.
the existing shareholders. There have been a number of companies who announced corporate actions leading to price adjustments during the period of study. These actions/announcements, as received by the NSE, are presented in Tables-2(a,b). On the basis of them daily close prices have been adjusted for such actions. Similar changes in market capitalization are also called for while constructing the new base value of the market index, S&P-CNX NIFTY from time to time, which, have presumably, been carried out by the NSE while providing the daily quoted values of the index.

Using the price adjusted data file(adjP_t), the corresponding returns variable(R_t) is generated for all fifty stocks during the study period according to the following formula.

\[ 1 + \frac{R_t}{100} = \frac{adjP_t}{adjP_{t-1}} \ldots (3.1) \]

Similarly, using the closing values of NIFTY index, the corresponding market returns variable(M_t), and wherefrom, \( (1 + \frac{M_t}{100}) \) is generated. One could use these variables and run a multiple regression of stock returns on the current market returns, and non-synchronous market returns variables, as per Dimson’s AC method, and obtain unbiased estimates of systematic risk. But instead, this study has applied Dimson’s AC method to the excess returns, which are basically stock (or market) returns in excess of a risk-free return. The 365-Day Treasury bill(TB), floated by the Government of India from time to time through the Reserve Bank of India, is used as a proxy for the risk-free asset, and the yield to maturity(YTM) implicit at the cut-off rate of 365-Day TB auctions is considered as the annual return on this asset. Table-3 gives the annual data on YTMs for the study period. The YTMs are first converted to daily basis, as represented by R_{ft}, and then used in deriving excess returns on stocks(Z_t) as well as excess returns on the market (EM_t), using the following formulae.

\[
\begin{align*}
\ln(Z_t) &= \ln(1 + R_t/100) - \ln(1 + R_{ft}/100) \\
\ln(EM_t) &= \ln(1 + M_t/100) - \ln(1 + R_{ft}/100)
\end{align*}
\]

4. Analysis of Results, Findings and Comparison

4.1 Analysis of Results

Unbiased estimates of systematic risk have been obtained for 50 stocks by running a multiple regression of excess stock returns on the synchronous and non-synchronous market returns as shown below, using the price adjusted data for the study period.

\[
\hat{\ln}(Z_t) = \alpha + \sum_{k=-n}^{n} \hat{\beta}_k EM_{t+k} + w_t \quad \text{and} \quad \hat{\beta} = \sum_{k=-n}^{n} \hat{\beta}_k
\]

Taking \( n = 1 \) and \( 5 \) alternatively, unbiased estimates of \( \beta \) have been obtained for all the 50 stocks constituting the market index and are presented in Table-4. As noted earlier, ten out of 50 stocks in the cross-section exhibited non-synchronous trading features during the period of study. A simple method of regression applied to the market model of stock returns (without any leads and lags) would yield biased estimates in the case of...
not only these ten infrequently traded stocks, but also the other 40 stocks which are constituents of the market index, and which are more frequently traded. To observe the effect of non-trading on bias in estimation, beta estimates have been obtained by (a) Simple method (without any lags or leads) (see col.2 of Table-4) (b) Dimson method with one lead and one lag incorporated into the model (col.3 of Table-4), and by (c) Dimson method with 5 leads and 5 lags included in the model (col.4 of Table-4) for individual stocks. On comparison of these methods of estimation, the following five important findings can be seen.

a) Of the ten so-called infrequently traded stocks, six shares reported lower values of betas as per simple model as compared to the corresponding beta estimates generated by Dimson method. viz., for Britannia, the beta estimate from simple method is only -.24 (much lower), but it has increased to -.10 when obtained from Dimson method (5 leads, 5 lags), and to 0 when Dimson (1 lead, 1 lag) method has been applied\(^\text{11}\). Similarly, for Hindustan Petroleum, the values of betas have improved from 0.74 (simple method), to 0.77 (Dimson (1 lead, 1 lag)) and to 0.95 (Dimson (5 leads, 5 lags)). Likewise, Infosys, Zee Tele, BHEL and MTNL, all seem to have suffered from a downward bias, or, their beta estimates are lower when simple method has been applied, as against the Dimson method, which corrects for such bias.

b) Of the remaining four infrequently traded stocks, NIIT and P&G stocks reported no change in beta values between the simple method or any of the two choices of Dimson methods of estimation, indicating, perhaps, that the bias problem is not serious in their cases. On the other hand, Hero Honda and Reckitt & Colman stocks have shown an upward bias, which could be due to the fact that trading frequency has undergone change over time. Moreover, they were found to have not traded for only one day during the whole study period.

c) There are 40 stocks which are relatively frequently traded. Of them, fourteen stocks reported downward bias in their betas. For five stocks, namely, Asian Paints, Bank Of India, Novartis, Reliance Petroleum, and SBI there has been no bias at all, and for the remaining 23 stocks, bias has been found to be upward, when a simple regression of market model was used as against Dimson method (with 1 lead and 1 lag). In the whole cross-section of 50 stocks, bias can be found to be downward for twenty stocks, upward for twenty six stocks, and negligible for four stocks only\(^\text{12}\).

\(^{11}\) A negative value of beta shows that Britannia share is defensive to market factors, but after correction for bias due to infrequent trading, the negative value turned to zero. It means that, of all, this stock can be considered as the least sensitive to changes in market factors on an average.

\(^{12}\) This finding of downward bias in estimation even for frequently traded stocks is with reference to our classification of stocks into ‘infrequently traded’ or ‘frequently traded’ categories. This may look contradictory to the general observations made by Dimson that bias is downward for infrequently traded stocks and upward for frequently traded stocks. It may be possible that the sample of 50 stocks comprising the NIFTY index, as considered in this study, is not sufficient enough to capture the effect of non-trading of a stock on the estimation bias under such classification. A larger cross-section of stocks, which could be inclusive of a more number of infrequently traded stocks, may have to be considered so as to arrive at any generalization about the nature and direction of the effect of non-trading on estimation bias.
d) The total weight of those stocks whose systematic risk was biased downward works out to be about 47.65 % of market capitalization as of July 31, 2000. Those stocks whose betas were biased upward make up about 38.3 % of market capitalization, while for the balance stocks, it comes to 13.5%.

e) However, estimation bias is minimized when Dimson method is applied to the cross-section of 50 index stocks. Two alternative combinations of leads and lags have been considered here for comparative purpose and to fix the number of leads and lags; they are a) one lead-one lag b) 5 leads-5 lags.

Between the two, which alternative should be preferred over the other depends on the true variance of betas estimated under the two alternatives. The one, which gives minimum variance would be more efficient and should, therefore be preferred. An estimate of variance of the true value of beta is given by the formula at 2.3 in section 2. It is obtained by cross-sectional variance of betas estimated minus the cross-sectional mean of variances of beta estimates for 50 stocks. In order to examine this, consider results given in Tables 5,6 and 7. Table 5 has two panels. Panel-1 presents results of beta coefficients estimated by Dimson method with 1 lead and 1 lag, which represent coefficients of 1) current returns on market index, denoted by B, 2) one-period lagged returns on market index, written as B(-1), and 3) one-period leading returns of market index, B(+1). For instance for ABB share, the values of beta estimates of B, B(-1) and B(+1) are 0.6763, 0.0410 and 0.0328, respectively. Between the two, which alternative should be preferred over the other depends on the true variance of betas estimated under the two alternatives. The one, which gives minimum variance would be more efficient and should, therefore be preferred. An estimate of variance of the true value of beta is given by the formula at 2.3 in section 2. It is obtained by cross-sectional variance of betas estimated minus the cross-sectional mean of variances of beta estimates for 50 stocks. In order to examine this, consider results given in Tables 5,6 and 7. Table 5 has two panels. Panel-1 presents results of beta coefficients estimated by Dimson method with 1 lead and 1 lag, which represent coefficients of 1) current returns on market index, denoted by B, 2) one-period lagged returns on market index, written as B(-1), and 3) one-period leading returns of market index, B(+1). For instance for ABB share, the values of beta estimates of B, B(-1) and B(+1) are 0.6763, 0.0410 and 0.0328, respectively. Their sum gives Dimson beta (1 lead, 1 lag), which is 0.7501. Likewise, for ACC, the Dimson beta value is 1.2247, and for Asian Paints, the unbiased estimate of beta is 0.5963, and so on.

Panel-2 gives similar beta coefficients when 5 leads and 5 lags of market returns variables are used in addition to the current market returns as regressors. For ABB again, the coefficient of current market returns, B is given by 0.6766, the lagged coefficients of beta are given by B(-1), B(-2), ---, B(-5) whose values are: 0.0396, 0.0017, 0.0288, 0.0218 and 0.0855 respectively; while the leading coefficients of beta are given by B(+1), B(+2), ... B(+5) whose values are 0.0407, -0.0107, 0.0333, -0.0171, -0.0126 respectively. According to the Dimson method with 5 leads and 5 lags, these 11 coefficients are aggregated to yield 0.8877, an unbiased estimate of the systematic risk of ABB.

For ABB, which estimate of systematic risk should be preferred- is it 0.7501 or 0.8877, both being unbiased ones? As discussed earlier, the answer lies in the rule (2.3) given earlier in section 2. It suggests that the one that has a lower variance of true beta coefficient should be preferred. An estimate of true variance of beta is obtained by finding the difference of the two components (a) and (b) below.

a) The cross-sectional variance of beta coefficients of 50 stocks under both alternatives are shown separately in two panels in column.2 of Table 7, while

b) the cross-sectional mean of variances of beta coefficients of 50 stocks, as estimated under the two alternatives; are shown separately in two panels in column.3, and their difference in column.4 of Table 7.

Consider the coefficient of current market returns, B that is estimated to have a true variance of 0.0171 under panel-1, when Dimson method with one lead and one lag is used. This is much lower than the corresponding value under the second alternative, which is 0.0267. Furthermore, as more number of lags are introduced, the cross-sectional variances of betas can be found to increase from 0.0577 to 0.0671 for B, from
0.0049 to 0.0052 for B(-1), whereas, they have remained constant for B(+1). As far as the cross-sectional mean of variances of betas is concerned, the respective values of B, B(-1) and B(+1) coefficients have remained constant around 0.0405 in both alternatives (see col.3 of Table 7), except for the beta coefficient, B(+3) in panel-2. The net difference of these two components (a) and (b), or, the difference of cols. 2 and 3 of Table 7, has thus resulted in a lower variance of true values of betas when estimated under Dimson method with one lead and one lag.

Nevertheless, for some of the infrequently traded stocks such as Britannia, Hindustan Petroleum, and MTNL, variances of beta coefficients are estimated to be smaller, though marginally, under panel-2 than their counterparts under panel-1. Overall, it appears that although both the alternatives of Dimson method yield unbiased estimates of betas, the one having one lead and one lag, seems to have lower variance. Hence it is more efficient than the second one and should be preferred.

4.2 Comparison of Dimson betas (unbiased) with Simple betas (biased) and Test of Significance of bias

It is contended that the systematic risk of a stock as assessed by the simple method of regression of a market model which incorporates only current period returns on the market index as a regressor will be biased even if some of the constituents of market index are infrequently traded (since the remaining majority of index stocks, though frequently traded, could also be biased). It may therefore be necessary to establish the significance of bias across the sample of stocks comprising the index. For this purpose, a comparison is made between the beta estimates obtained by Dimson (one lead, one lag), called ‘Dimson betas’ and those by Simple method, called ‘simple betas’ using the same data set of fifty index stocks. Tables 8A and 8B present the comparative results. Table 8A gives the estimates of Simple Betas (col.2) and Dimson betas (col.3), and the bias, which is the value of Dimson beta minus Simple beta (col.4). In order to test whether or not this bias is significant, a test statistic (T) is constructed as follows:

\[
\beta_{\text{DIM}} - \beta_{\text{SIM}}
\]

The statistic to test the null hypothesis, \(H_0\)

\[
\beta_{\text{DIM}} - \beta_{\text{SIM}} \approx \frac{\sqrt{\hat{\beta}^2_{\text{DIM}} + \hat{\beta}^2_{\text{SIM}}}}{\text{S.E.}}
\]

Since \(\hat{\beta}_{\text{DIM}} = \hat{\beta} + \hat{\beta}, \) it follows that

---

13 It may be argued that as NIFTY constituents are not generally infrequently traded, the extent of bias of out of 50 stocks of the index have shown relatively low frequency of trading, while the rest of the index stocks are highly liquid in general. However, beta estimation of index stocks by simple market model method would not yield unbiased estimates as found in this paper. Furthermore, as a critic noted in the is not the right choice to study the problem of non-synchronous trading since there are not many infrequently traded stocks in this cross section. The Dimson methodology to Indian stocks, but to assess the systematic risk of index stocks, given some extent of infrequent trading among them.
\[ \sigma^2_{\text{DIM}} = \text{Var}(\hat{\beta}) + \text{Var}(\beta_{-1}) + \text{Var}(\beta_{+1}) + 2 \text{Cov}(\beta, \beta_{-1}) + 2 \text{Cov}(\beta, \beta_{+1}) + 2 \text{Cov}(\beta_{+1}, \beta_{-1}) \]

\( \sigma^2_{\text{SIM}} \) is the variance of simple beta, \( n_1 \) and \( n_2 \) are the number of observations used in estimating Dimson beta, and simple beta respectively. Variances of Dimson betas have been obtained for each of fifty stocks using the estimates of respective variance-covariance matrices and are shown in Table 8B, which in turn are used to evaluate the test statistic as shown under columns (5) to (7) of Table 8A. It can be seen that bias is statistically significant at 1% level as the t-values are quite high in as many as 46 out of 50 stocks considered. In the case of four stocks where bias is not significant there is almost no difference between the pairs of values of systematic risk of a stock derived by the two methods. It amply suggests that estimates of systematic risk obtained by simple method are biased considerably, and this finding points to a technical improvement in the results obtained by Dimson methodology in this paper.

### 4.3 Interpretation of Dimson Betas, and their Implications

Finally, unbiased estimates of systematic risk thus obtained are used to make a comparative analysis and also to find out which stocks are yielding greater excess returns per unit risk in the Indian stock market. Table-8C reports Dimson betas (1 lead, 1 lag) of individual stocks along with their average daily returns, volatility and weights in the index according to market capitalization. For convenience, the individual stocks are classified into two types. A) More volatile stocks, which have reported a Dimson beta value of at least one and B) Less volatile stocks, which have reported a Dimson beta value of less than one. For analytical reasons, Sharpe's measure of excess returns per unit risk is considered, which is given by the ratio of excess returns to the risk of a security. For getting this, first excess returns are obtained for each stock by subtracting daily average yield on the 365 days T-bill (see Table-3) from the actual stock returns on a traded day, and then average excess returns for the period is derived. This is as shown for 50 stocks in column 6 of Table 8C. Later the average excess return per unit risk is computed by dividing the average excess returns of a stock by its standard deviation, and then annualised by multiplying with 365. This is shown in column 7 of Table 8C. Cross correlations have been computed between betas, volatility, and average daily excess returns for the cross-section of index stocks. The results are shown at the end of Table 8C.

Comparatively, the market index, NIFTY-50, is found to be heavily loaded with more volatile stocks, their combined weight being 54% of the total market capitalization represented by the index, although they number only 19 out of 50. But of them, excess returns per unit risk was more than 10% per annum only in the case of seven stocks. It is worth noting that Infosys Technology tops the list with the highest excess return per unit risk at 48.89% per annum, followed by Satyam Computers at 42.68%, Zee Telefilms (30.59%), D.r.Reddy (17.25%), while others gave less than 15% per annum on an average.

As for less volatile stocks, there are nine well-known and frequently traded stocks which have yielded a respectable excess return per unit risk (see B-part of Table 8C) of at least 15% per annum. Of them, NIIT gave highest return of 28.7% per annum, followed by Hero Honda and HDFC Bank, each at 22.4%, CIPLA (21.9%), Hindustan Lever (17.8%), Britannia (16.3%), Dabur (13.7%) and Reliance Petroleum (11.1%) etc. This finding indirectly implies that returns are not commensurate with risk for a number of
highly volatile stocks, and there are decent returns on an average from less volatile stocks in the market. It is, therefore, necessary to be highly selective while designing a portfolio of stocks, and the beta estimates derived in this study should give useful insight into portfolio selection by investors at large, and portfolio managers in particular.

More importantly, a careful examination of the cross-correlations computed suggests that Dimson betas derived in this study are closely associated with volatility of stock returns, as the coefficient of correlation comes out to be 0.75 between them. The total risk of a security as given by its volatility varies directly and closely with the market risk of that security on an average in the cross-section of companies considered. The daily excess returns are weakly associated with systematic risk (betas), since the cross-correlation coefficient is estimated to be 0.19 only. Using data on excess returns and beta of 50 index stocks as given in Table-8c, a simple regression of excess returns on betas for a sample of 50 stocks has been worked out. The cross-sectional regression equation of the security market line in the Indian context is shown below. It is clear from the equation that the relationship between risk (as measured by beta) and the excess returns of Index stocks appears to be rather weak, and the security market line seems to be flat, as the slope coefficient is statistically insignificant though positive.

\[
\text{ExcessReturns} = 0.652 + 5.54 \text{ Beta} \\
\quad (0.091) (0.722)
\]

R-Squared = 0.012; R-Bar-Squared= .03; n =50
(Figures in parentheses are the respective t-ratios)

To analyse this more in detail, one may have to examine the Capital Asset Pricing model (CAPM) in a different framework taking into account fluctuations of risk and return over a longer period of time which is beyond the scope of this study. It is thus not possible to make a firm conclusion about the validation of CAPM hypothesis in the Indian stock market on the basis of this study.

5. Summary, conclusions and limitations of the study

This study has dealt with the econometric estimation of systematic risk of a cross-section of 50 stocks that constitute the market index, namely, S&P-CNX NIFTY. As some of them are infrequently traded during their trading history on the National Stock Exchange, a simple regression of market model yields biased estimates of betas. Since betas are used in practice for financial and investment decisions, unbiased estimation of betas is essential for a prudent decision making process. Hence this study was undertaken. In the literature, various authors have examined the problem of infrequent trading extensively. Among them, Dimson’s aggregate coefficients method has been found to be useful and adopted to the Indian stock market in this study. Most important findings of this study are as follows.

1. **Dimson method has yielded unbiased estimates for all 50 stocks of NIFTY.** In comparison, similar estimates of risk obtained by simple market model turned out to be significantly biased in a majority of cases.

2. **Out of many Indian stocks, which are frequently traded and also volatile, only a few have been found to yield good excess returns per unit risk.** The average of daily
excess returns were not commensurate with risk taken while investing in many volatile stocks, barring a few leading scrips, like Infosys Technology, Satyam Computer Services, Zee Telefilms, NIIT, Hero Honda, HDFC Bank, CIPLA, Hindustan Lever and Britannia, for which the excess returns per unit risk exceeded 20% per annum on average during the study period, November 3, 1995 to May 23, 2000.

3 The unbiased estimates of systematic risk obtained in this study revealed which stocks are more volatile and which are less volatile than the market in general.

4 The ranking of stocks by biased beta values as per the simple regression method was much different from what it is due to Dimson method, which gives unbiased estimates of betas. This has very important implications for practical considerations in financial investment in stock market, hedging, index futures trading as well as for corporate finance decisions in real sectors.

5 The direction of bias is not specific to the trading frequency of stocks. In particular, bias is not downward in general for infrequently traded stocks, or upward in general for more frequently traded stocks. As discussed, this finding seems to contradict Dimson’s argument about the relationship between trading frequency and direction of bias. It is contended that this study has used a sample of stocks that constitute NIFTY-50 index, which have some infrequently traded stocks in their trading history, but this sample of stocks is not exhaustive enough to verify such pattern of relationship, and hence no such generalization can be made from our results.

In conclusion, it may be pointed out that the unbiased estimates of betas obtained in this study indicate that there seems to be a strong possibility for the existence of a weak relationship between risk and return in the Indian stock market, and that portfolio managers of Indian stock market may make note of results of this exercise for a prudent decision making.

As a limitation, it should be pointed out that there is every possibility for beta values to undergo changes from time to time on account of changes in the trading frequency of stocks, market factors including regulatory changes in the stock market. All of them might cause instability in beta estimates, and hence, there is a need to evaluate them from new data sets after a lapse of time. There is, therefore, a need to caution the investing community to keep this limitation in mind before using these values for practical purposes of portfolio management as also hedging. Secondly, this study has made an attempt to find cross-correlations between betas and volatility and excess returns of a stock. It observed that the degree of correlation between risk and return is about 0.19. But this relationship needs to be examined more in detail by taking a larger sample of stocks over a longer period of time. An important outcome of this study is that unbiased estimation of betas has a bearing on the choice of a larger cross-section of stocks, and the market index rather than a limited sample.
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