Extreme Value Volatility Estimators and Their Empirical Performance in Indian Capital Markets

Ajay Pandey[?]

Abstract

Despite having been around for a long time in the literature, extreme-value volatility estimators have not been used extensively. The extreme-value estimators take into account the highest and lowest prices observed during the day besides the opening and closing prices. The extreme-value estimators have not been used, as they are likely to be biased in case the strict assumptions, under which they were derived, are invalid. Recent research has however, shown that the traditional volatility measures (sample standard deviation of returns) could also be biased under certain return generating processes. The performance of extreme-value estimators therefore, needs to be evaluated empirically. In this study, we use three-years' high-frequency data set of five-minutes returns to construct measures of realized volatility with which some of the extreme-value estimators proposed in the literature and the traditional estimators are compared. Based on five criteria used to evaluate the bias, efficiency and predictive power, we find that almost all the extreme-value estimators are free of bias and perform well compared to their traditional counterparts for the S&P CNX Nifty stock-index and the 10 constituent stocks studied. We also find that the extreme-value estimators are 2-5 times more efficient and have better predictive power. With the exception of the Parkinson estimator for the index, all are unbiased. Even though specific estimators perform well for a particular asset, all the estimators perform well enough to justify their use when compared with the traditional estimators. The efficiency gains are however, marginal in case of relatively illiquid stocks.

[?] Ajay Pandey is a member of faculty at Indian Institute of Management, Ahmedabad-380015, India.

We would like to thank the National Stock Exchange for the research funding as part of its "Research Initiative" and also for providing high frequency data to estimate the realized volatility. We are also thankful to the two anonymous referees for their comments on the proposal, which helped in improving the research design. The views expressed in this paper are of the author and not necessarily of NSE.

1. Introduction

Volatility estimates are used extensively in empirical research, risk management and derivative pricing by the finance professionals and researchers. Traditionally, volatility of asset returns has been estimated using sample standard deviation of close-to-close daily returns and is scaled to estimate volatility for any period (such as annual, monthly etc.). Following work by Parkinson (1980), numerous extreme value (or range based) estimators have been suggested in the literature. Theoretically, the extreme value estimators are shown to be more efficient (5 to 14 times), yet they have not been very popular. This is mainly because these estimators are derived under strong assumptions about underlying returns generating process in the asset markets. It is assumed that the asset prices follow geometric Brownian motion (GBM) and are observable in a market trading continuously. While the extreme-value estimators of volatility could be biased if the returns generating process is mis-specified, Li and Weinbaum (2000) point out that the assumed "unbiasedness" of the traditional estimator itself, is contingent on the validity of particular return generating process. In particular, they show that the traditional estimator based on the sample standard deviation/variance of returns is not an unbiased estimator of the true instantaneous volatility/ variance for the trending Ornstein-Uhlenbeck process having predictable returns and constant volatility. They argue that the bias in the traditional or extreme-value estimators is more of an empirical issue. It is possible however, to assess the efficiency and/or bias of the traditional and extreme-value estimators of volatility using high frequency data. As shown by Andersen et al. (2001a, 2001b), estimates of the realized volatility calculated using high frequency data are model-free under very weak assumptions. The realized volatility estimates can be used to test the bias and efficiency of different types of the extreme value volatility estimators empirically.

Volatility modeling and forecasting in last 10-15 years has spawned extensive theoretical and empirical research on "Conditional Volatility", following the pioneering work of Engle (1982). It has been empirically found that the volatility in financial markets is not constant. As a result, various parametric models have been proposed in attempts to model intertemporal behavior of volatility. As opposed to the time-varying characteristics of the observed volatility, the traditional and extreme value estimators attempt to measure or estimate constant or "Unconditional Volatility". One reason for relatively less research on the extreme value estimators could be that the volatility is widely acknowledged to be varying with time. Even if the volatility in the financial market is time varying, use of the extreme-value estimators may still be preferred if they are as efficient empirically as implied by the theory. In that case, conditional volatility models, efficient & parsimonious extreme value volatility estimators and high frequency data based realized volatility model could possibly compete for modeling and forecasting volatility for various applications.

In this paper, we empirically examine the properties of some important extreme value estimators suggested in the literature vis-à-vis the realized volatility measure in the context of Indian Capital Markets. This analysis is similar to a study by Li and Weinbaum (2000) in US context, in which they investigated the performance of extreme value estimators for two stock indices (S&P 500 and S&P 100), a stock index futures (on S&P 500) and three exchange rates (Deutsche Mark: US$, Yen: US$ and UK Pound: US$). One of the reasons for a straightforward replication of the Li and Weinbaum study in Indian context is the result obtained by them. While they found overwhelming support, in terms of bias and efficiency, for the extreme value estimators in case of stock indices, the performance of extreme value estimators for other assets was less clear. The study reported that the extreme value estimators were biased estimates of the realized volatility despite being efficient. The biasedness and efficiency of the extreme-value estimators therefore, remains an empirical issue for their potential use in any specific asset market. Besides empirically investigating the performance in Indian context, we also extend the analysis in two important ways. Firstly, since most of the extreme value estimators assume driftless price dynamics, we report the empirical performance of these estimators for various sub-periods within the sample period in order to examine their biasedness and efficiency when the asset prices may exhibit substantial drift. Secondly, the extreme value estimators suffer from "discrete trading" induced bias and loss of efficiency in case the underlying asset is not trading continuously (Marsh and Rosenfeld 1986, Cho and Frees 1988). The induced bias and loss of efficiency may be less severe for the indices than for the assets, which are less frequently traded. Besides analyzing empirical performance of the extreme value estimators for the chosen stock index (S&P Nifty), we investigate performance of the extreme value volatility estimators for 10 constituent stocks having different volatility and liquidity characteristics. In the next section of this paper, we review various extreme-value estimators proposed in the literature. We also report the theoretical and empirical results on the performance of the extreme-value estimators. In section three of the paper, we describe the data set and methodology used in the study of empirical performance of the extreme-value estimators. In section four, we report the empirical results of this study. We conclude in section five by summarizing the results and pointing the direction for future research in the Indian context.

2. Review of Research on Extreme-Value Estimators

2.1 Unconditional Volatility and Extreme-Value Estimators

Traditionally, the unconditional volatility of asset returns has been estimated using close-to-close returns. The traditional close-to-close volatility (or, variance) estimator (?cc) for a driftless security is estimated using squared returns and is given by-

$$?_{cc}^2 \quad = \quad 1/n\ ?\ (c)^2 \qquad \text{... ... .} \quad (1)$$

where,

n = Number of days (or, periods) used to estimate the volatility

c = $\ln C_t - \ln C_{t-1}$

C_t = Close price of day t

The mean-adjusted version of the close-to-close estimator ($?_{acc}$) is estimated using sample standard deviation and is given by-

$$?_{acc}^2 \quad = \quad 1/(n-1)*[\ ?\ (c)^2 - n\underline{c}^2/] \qquad \text{... ... .} \quad (2)$$

where,

$\underline{c}$ = $(\ln C_n - \ln C_0)/n$

Parkinson (1980), following the work of Feller (1951) on the distribution of the trading range of a security following geometric Brownian motion (GBM), was first to propose an extreme-value volatility estimator for a security following driftless[1] GBM, which is theoretically 5 times more efficient compared to traditional close-to-close estimator. His estimator ($?_p$) is given by-

$$?\,^2_p \quad = \quad 1/(4n \ln 2)*\ ?\ (\ln H_t/\ L_t)^2 \qquad \text{... ... .} \quad (3)$$

where,

H_t = Highest price observed on day t

L_t = Lowest price observed on day t

Extending his work, Garman and Klass (1980) constructed an extreme-value estimator incorporating the opening and closing prices in addition to the trading range, which is theoretically 7.4 times more efficient than its traditional counterpart. Their estimator ($?_{gk}$) is given by-

$$?\,^2_{gk} \quad = \quad 1/\ n *\ ?\ (0.511(\ln H_t/\ L_t)^2 - 0.019(\ln (C_t/\ O_t\ )*\ln (H_t\ L_t/\ O_t^2)$$
$$- 2 \ln (H_t/\ O_t)* \ln (Lt/\ O_t)) - 0.383(\ln C_t/\ O_t)^2) \qquad \text{... ... .} \quad (4)$$

where,

O_t = Opening price observed on day t

[1] Driftless means that log price process is driftless, i.e., ? = ?²/2. The process is specified as $dS_t = ?\ S_t\, dt + ?\ S_t\, dW_t$, where W_t is a standard Brownian motion and S_t is the price of asset at time t.

Both the Parkinson and Garman-Klass estimators despite being theoretically more efficient are based on assumption of driftless GBM process. Rogers and Satchell (1991) relaxed this assumption and proposed an estimator ($?_{rs}$), which is given by-

$$?^2_{rs} \quad = \quad 1/n * ? (\ln (H_t/C_t) \ln (H_t/O_t) + \ln (L_t/C_t) \ln (L_t/O_t)) \quad \ldots \ldots . \quad (5)$$

Kunitomo (1992) also proposed an extreme-value estimator based on the range of a Brownian Bridge process constructed from price process, which is 2 times more efficient than Parkinson estimator. His estimator however, cannot be computed directly from the daily data. Later, Spurgin and Schneeweis (1999) proposed an estimator based on the distribution of the range of Binomial Random walk. Their estimator ($?_{ss}$) is given by-

$$?^2_{ss} \quad = \quad 1/n^2 * 0.3927 * ? (\ln H_t/L_t)^2 - 0.4986\, S \quad \ldots \ldots . \quad (6)$$

where,

S = The tick-size of the trades

Recently, Yang and Zhang (2000) proposed an estimator independent of drift, which also takes into account an estimate of closed market variance. The estimators proposed earlier, including the Rogers-Satchell estimator, do not take in to account the closed market variance. This means that the prices at the opening of the market are implicitly considered same as that of closing price on the previous day. The Yang-Zhiang estimator is based on the sum of estimated overnight variance and the estimated open market variance. The estimated open-market variance in turn, is based on weighted average sum of the open-market return sample variance and the Rogers-Satchell estimator with the weights chosen to minimize the variance of estimator. The Yang-Zhiang estimator ($?_{yz}$) is given by-

$$?^2_{yz} \quad = \quad 1/(n-1) * ? (\ln O_t/C_{t-1} - \underline{o})^2 +$$

$$k/(n-1) * ? (\ln C_t/O_t - \underline{c})^2 + (1-k) * ?^2_{rs} \quad \ldots \ldots . \quad (7)$$

where,

$\underline{o}$ = $1/n * ? (\ln O_t/C_{t-1})$

$\underline{c}$ = $(\ln C_n - \ln C_0)/n$ or, $1/n * ? (\ln C_t/O_t)$

k = $0.34/(1.34 + (n+1)/(n-1))$

2.2 Extreme-Value Estimators: Theoretical Issues

The extreme-value estimators proposed in the literature have been usually derived under strong assumptions. As pointed out in the previous section, attempts have been made to relax the assumption of driftless price process and closed market variance by Rogers and Satchell (1991) and Yang and Zhang (2000) respectively. Besides these, it is argued that the observed extreme values may reflect certain liquidity-motivated trades (Li and Weinbaum 2000). This could make them less representative of "true" prices as compared to the closing prices.

Besides extreme values being potentially less representative of true prices, the extreme values are observed in the markets, where the trading is discrete. The extreme-value estimators on the other hand, are derived under assumption of continuous trading. This can induce downward "discrete trading" bias in the extreme-value estimators, as the observed highest prices are lower than the "true" highest price and observed lowest price is higher than the "true" lowest price (Rogers and Satchell 1991, Li and Weinbaum 2000). Rogers and Satchell (1991) addressed this issue by proposing adjustment in the extreme-value estimators, which takes into account number of steps (trades) explicitly. The adjusted Rogers-Satchell estimator ($?_{ars}$) is positive root of the following equation-

$$?^2_{ars} = (0.5594/N_{obs})* ?^2_{ars} + (0.9072/N^{1/2}_{obs})* \ln (H_t/L_t)* ?_{ars} + ?^2_{rs} \quad \ldots \ldots . \quad (8)$$

where,

N_{obs} = Number of observations/ transactions

$?_{rs}$ = Unadjusted Rogers-Satchell Estimator

Rogers and Satchell also proposed similar correction to the Garman and Klass (1980) estimator. The adjusted Garman-Klass estimator ($?_{agk}$) is positive root of the following equation-

$$?^2_{agk} = 0.511*[(\ln H_t/L_t)^2 + (0.9079/N_{obs})* ?^2_{agk} + (1.8144/N^{1/2}_{obs})*\ln H_t/L_t*?_{agk}] + 0.038*[\ln H_t/O_t* \ln L_t/O_t - (0.2058/N_{obs})* ?^2_{agk} - (0.4536/N^{1/2}_{obs})* \ln (H_t/L_t)*?_{agk}] - 0.019* \ln (C_t/O_t)*\ln (H_t L_t/O_t^2) - 0.383*(\ln C_t/O_t)^2 \quad \ldots \ldots . \quad (9)$$

2.3 Extreme-Value Estimators: Empirical Results and Issues

The extreme-value estimators proposed in the literature have been tested using simulated stock prices, actual stock prices and recently, using realized volatility measures. Garman and Klass (1980) using simulated data with discrete price changes, show that extreme-value estimators are downward biased. Beckers (1983) using actual data also found downward bias in extreme-value estimators. Studies by Wiggins (1991, 1992) also reached similar conclusions. However, Spurgin and Schneeweis (1999) found that the binomial estimator developed by them outperformed traditional and other extreme-value estimators on daily and intra-day day data of two futures - CME SP500 and CBT Treasury Bonds contracts. Li and Weinbaum (2000) using intra-day high frequency data to measure realized volatility, found overwhelming support for the extreme-value estimators for stock indices (S&P 500 and S&P 100) data set, but confirmed bias of the extreme-value estimators for currencies and S&P 500 futures data set despite efficiency gains. Their results are important in pointing out that the results of preceding studies showing biasedness of the extreme value volatility estimators are not valid, at least for some asset markets. They point out that the traditional close-to-close volatility estimator could as well be seriously biased and hence the reported downward bias in previous studies could be due to upward bias of the close-to-close volatility estimator. If there is no trade-off between bias and efficiency, then use of the extreme value estimators by practitioners, regulators etc. may be clearly superior due to efficiency gains. The Li and Weinbaum study attempts at resolution of this assumed trade-off. This was made possible by the use of "realized volatility" as benchmark using high frequency data, traditionally not used by or available for such studies.

Given the arguments and findings of Li and Weinbaum study, empirical performance of the extreme-value estimators remains an open issue. In applications where traditional estimators are still being used and relied upon, the extreme-value estimators could provide effective alternatives if they are unbiased and more efficient. In this study, our focus is limited to this issue. Any comparison with the competing "Conditional Volatility" and "Realized Volatility" models for volatility modeling and forecasting remains outside the scope of this work. Theoretically at least, if the efficiency gains from extreme-value estimators are high, they could also prove to be effective in forecasting time varying volatility. However unlike the other models, the extreme-value estimators proposed in the literature do not model full variance-covariance matrix of asset returns and this itself limits their scope of application.

3. Research Methodology

In this study, our objective is to empirically investigate the performance of some of the extreme-value estimators proposed in the literature. With the availability of high-frequency data being compiled by the National Stock Exchange, a direct comparison of estimates with the model-free realized volatility estimates is possible and hence the realized volatility estimates have been used in the study to assess the bias and efficiency of the extreme-value estimators. The traditional close-to-close estimators are also computed and compared with the realized volatility estimates. In this section, we describe the data set used, the measure of realized volatility and discuss the performance criteria used to assess the performance of extreme-value estimators.

3.1 Data

In this study, we use 11 sets of high-frequency data. Of these, one is on S&P CNX Nifty, a stock index of National Stock Exchange, Mumbai, based on 50 large capitalization stocks. The other 10 sets are of individual stocks, which are constituents of this index. The stocks are chosen with a view to have a diverse set of stocks in terms of volatility and liquidity characteristics. Even though no formal criteria are applied for the selection of these stocks from within the constituents of the index, the stocks selected exhibit the expected diversity during the period under study. Table 1 gives summary of the average volume and the realized daily volatility associated with these stocks. As the National Stock Exchange started compiling the high-frequency data for research purposes since 1999, our data set covers the period of January 1999- December 2001, i.e., three years. NSE records the data on the index for each day separately, whereas for the stocks we extract it from the file containing all the trades during the day. Of the 10 stocks chosen, three stocks are relatively illiquid (Novartis, Indian Hotel and SmithKline Beecham) during the period of the study. All the three stocks are moderately volatile during the period. Of the remaining seven stocks selected, two are liquid and less volatile (Hindustan Lever and Reliance), two liquid and moderately volatile (Larsen & Toubro and Infosys) and three liquid and highly volatile (Satyam, NIIT and Zee Telefilm).

Insert Table 1 about here.

3.2 Open Market Variance Estimates

As pointed out elsewhere in the paper, the extreme-value estimators prior to Yang and Zhiang (2000) did not take the closed-market variance (between the closing prices of the previous day and opening prices) into account. In the absence of any observation, the treatment of the closed-market variance however, has to be alike irrespective of any specific estimator being used for estimating the open-market variance. In order to compare, the traditional close-to-close estimator needs to be modified for estimating the open-market variance. We use accordingly, the modified open-to-close traditional estimator by replacing closing price of the previous day, C_{t-1} with opening price of the day t, O_t in equation 1. Similarly, the Yang-Zhiang estimator is modified to exclude the first term (of R.H.S.) related to the closed-market variance in equation 7. The open-market variance estimates of the traditional and extreme-value estimators are used for comparison with the realized volatility. In this study, we compute the Parkinson, Garman-Klass, Rogers-Satchell and Yang-Zhiang estimators for comparison. We use only the first three estimators for studying their performance on the individual stocks.

3.3 Realized Volatility Measurement

If high-frequency data is available, the volatility becomes observable and does not remain latent. The realized volatility measure developed by Andersen et al. (2001a) was used by Li and Weinbaum (2000) to directly compare performance of the extreme-value estimators. The realized volatility measure for day t is given by-

$$?^{2}_{t} = ?\ r^{2}_{j,t}$$

where,

$r^2_{j,t}$ = Squared return series of intra-day data

j = Intra-day interval over which returns are being measured

It is possible to annualize the realized volatility for any given day by scaling it up with an annualizing factor. The annualizing factor is simply square root of number of trading days in a year. Measuring realized volatility requires choosing appropriate interval over which the squared returns are used to measure the realized volatility. While shorter time intervals reduce the measurement error, they are also likely to be biased by the microstructure effects (Andersen and Bollerslev 1998, Andersen et al. 1999). Andersen et al. (2001a, 2001b) and Li and Weinbaum (2000) found that sampling the returns over 5-minutes interval is optimal. Without investigating the desirability of using 5-minute returns series on our data set, we have used it to compute the realized volatility.

3.4 Performance Criteria

In order to compare the bias and efficiency of the traditional and extreme-value estimators, the following finite sample criteria are used-

1. Bias of the Estimator
2. Mean Square Error of the Estimator
3. Relative Bias of the Estimator
4. Mean Absolute Difference of the Estimator
5. Mean Square Error of the Estimator in Forecasting Volatility one-period ahead

Of these, except the last one, all others are either standard measures or have been used by the Li and Weinbaum study. The first and the second criterion measure bias and efficiency respectively and are standard measures. The third criterion is to assess the magnitude of bias with respect to the true parameter (the realized volatility measure, in this case) as the first criterion gives only absolute amount. The fourth one is another measure of efficiency like the second criterion and is less affected by the outliers in the data set. The fifth and the last one indicates the efficiency of the estimator in forecasting the true parameter (realized volatility) one-period ahead. Since h-period volatility estimates of "unconditional volatility" are typically used in forecasting volatility h-period ahead, we include this criterion to evaluate the ability of the extreme-value estimators to forecast.

If the true volatility (realized volatility) on day t is $?_t$ and the estimated volatility given by an estimator is $?_{est}$, then the five performance criteria are computed as under-

Bias = $E\,(?_{est} - ?_t)$

Mean Square Error = $E\,[(?_{est} - ?_t)^2]$

Relative Bias = $E[(?_{est} - ?_t)/\ ?_t]$

Mean Absolute Difference = $E[Abs(?_{est} - ?_t)]$

MSE of one-period forecast = $E\,[(?_{est} - ?_{t+1})^2]$

4. Empirical Results

4.1 Descriptive Statistics for Realized Volatility

In case of the realized volatility measures for the chosen index (S&P CNX Nifty) and the 10 constituents stocks, two problems were faced in the measurement. Firstly, there have been trading breaks on quite a few days of trading at NSE because of communication and operational reasons. Since the extreme-value estimators and the traditional estimator are based on extreme values and closing prices reported for the entire day, we use the squared return series even if there are breaks. In other words, the returns between the breaks are treated as if they are 5-minute returns. A similar kind of problem arises for the stocks, which are relatively less liquid. This is likely to introduce measurement errors in the realized volatility measure and make them slightly downward biased. However, the bias introduced in the realized volatility measure need not affect the comparison between the traditional and extreme-value estimators. Secondly, the closing prices given by the National Stock Exchange are not the last traded prices, we use the last traded prices for measuring both the realized volatility and the volatility estimators for the stocks.

The descriptive statistics for the realized volatility is reported in Table 2. It reports the mean realized volatility (annualized), the standard deviation and the skewness and kurtosis measures. The mean value of the annualized realized volatility for the index is 26.02% during the period of the study. In case of stocks, it ranges from a low of 35.39% for Hindustan Lever to a high of 70.48% for Zee Telefilm. The standard deviation of the realized volatility seems to be correlated with the mean realized volatility of the stock. The skewness and kurtosis measures for the realized volatility across the index and stocks indicate substantial departure from distribution characteristics of normal random variables.

Insert Table 2 about here.

4.2 Extreme-Value Estimators: Results for the Index

The traditional as well as the extreme-value estimators are computed for non-overlapping periods of one-day, five-day and for each calendar month of the period under study. These volatility estimates are then compared with the corresponding period measures of the realized volatility. The performance is assessed using the five performance criteria discussed elsewhere in the paper. The results are reported in Table 3. Panel A of the table is for the estimates based on one-day period, panel B for estimates over non-overlapping five-day period and panel C for volatility estimated over one-month period. For the S&P CNX Nifty, the results overwhelmingly support use of the extreme-value estimators over the traditional estimator.

Insert Table 3 about here.

Of the four extreme-value estimators used in the study, all except Parkinson estimator exhibit no significant bias and in fact have lower bias than the traditional estimators for one-day and five-day estimates. Though not significant, the average bias for all the estimators across different estimation periods has negative sign. The average bias, when annualized, for one-day estimation period is –5% for the traditional estimator, -9.6% for the Parkinson estimator and about –2.4% for the Garman-Klass and Rogers-Satchell estimators. The Parkinson estimator exhibits larger negative bias consistently across estimation periods. It also exhibits smallest variance in the estimates across the estimation periods. For the five-day estimation period, the average bias, when annualized, is –1.2% and –1.4% for the traditional and adjusted traditional estimators. For the Parkinson estimator, it remains high at –4.5% while for the Garman-Klass, Rogers-Satchell, and Yang-Zhiang estimators, it is –1.1%, -0.9% and –1% respectively. For the estimation period of one-month, even though the computed bias of the traditional estimators is the least, it is not substantially different from the other three extreme-value estimators (Garman-Klass, Rogers-Satchell and Yang-Zhiang estimators). Except for the Parkinson estimator with average annualized bias of –2.3%, the average bias in case of all the other estimators is below –0.6%. The average bias in case of all estimators comes down with the increase in the length of estimation period, a result seen in case of individual stocks also. All these three extreme-value estimators also perform well on both the efficiency criteria with Yang-Zhiang estimator being the best performer. The gains in terms of efficiency however, range between 2-4 times depending upon the horizon. The gains are higher when the estimation period is shorter. This efficiency gain, even though less than what is theoretically implied, is still quite useful as the results indicate that the extreme-value estimators are not biased. The extreme-value estimators also perform well compared to the traditional estimators in forecasting the volatility one-period ahead across the estimation periods.

In order to examine the performance of the volatility estimators during various sub-periods of the study, we also report the analysis performed on one-day estimates for each of the three years of study in Table 4. This table also reports the analysis of five-day estimates for each of the three years. Similar to aggregate results, all the extreme-value estimators outperform their traditional counterparts in each of the three years of the study. In fact for one-day estimates, we find (not reported in this paper) that the extreme-value estimators outperform the traditional estimators in each quarter of the study in terms of bias as well as efficiency.

Insert Table 4 about here.

4.3 Extreme-Value Estimators: Results for the Stocks

Of the ten stocks analyzed in the study, we separately report the performance of extreme-value estimators' for- (a) Liquid and relatively less volatile stocks, (b) Liquid and relatively more volatile stocks, and (c) Relatively Illiquid stocks, respectively in panel A, B and C of Table 5. The first panel consists of the stocks of Hindustan Lever, Reliance Industries, Larsen & Toubro and Infosys Technologies Ltd. Panel B consists of the stocks of Satyam Computers, NIIT and Zee Telefilm Ltd. Panel C consists of the stocks of Indian Hotel, SmithKline Beecham Consumer and Novartis Ltd. A relatively small sample of stocks was grouped this way to evaluate whether the liquidity and volatility characteristics seem to affect the performance of volatility estimators.

In the case of four liquid and relatively less volatile stocks included in panel A, all the three extreme-value estimators used (Parkinson, Garman-Klass and Rogers-Satchell), perform well in terms of both bias and efficiency irrespective of estimation horizon. They are 3-5 times more efficient without any significant bias. Though insignificant, the average bias are relatively high for shorter estimation-periods (-5% to –9% annualized for the traditional estimator and –0.7% to –4.3% for the extreme-value estimators estimated over one-day) as compared to longer estimation period (all the estimators exhibit average bias of less than –1% annualized). The extreme-value estimators also predict volatility one-period ahead better compared to their traditional counterparts. The efficiency gains are more for shorter estimation periods (one-day and five-day). In this group, the Parkinson estimator turns out to be the best both in terms of bias and efficiency across estimation periods. Though the efficiency gains seem to vary with the estimator, all the three estimators are more efficient than their traditional counterparts without any significant bias.

In the next group of three relatively more volatile but liquid stocks included in panel B, the extreme-value estimators perform well in terms of bias, efficiency and prediction. The average bias of the estimators over one-day estimation period, though insignificant, is high. For the traditional estimator, when annualized, it is –5.2% for Satyam to –10.2% for Zee; in case of the extreme-value estimators, it varies from –0.8% for Satyam (the Parkinson estimator) to –7.5% for Zee (the Rogers-Satchell estimator). With the increase in the length of estimation period, it becomes lesser and is less than -1% for the estimators, except for the traditional estimators in case of Satyam (around +1.3%). The efficiency of the best extreme-value estimator is as about 6 times higher and on an average about 4 times, except for Zee Telefilms over monthly estimation period, when it drops to about 2 times. Like in the previous group of stocks, the Parkinson estimator performs well on all performance criteria across the estimation periods and stocks.

The last group of stocks reported in panel C consists of relatively illiquid stocks. Since extreme-value estimators are known to be sensitive to "discrete-trading" bias, the performance of extreme-value estimators in this group of stocks is of particular interest. Even though the results for the shorter estimation periods are similar to the other two groups, the extreme-value estimators are comparatively less efficient (only about two times) than their traditional counterparts. For the volatility computed over a calendar month, the extreme-value estimators are about as efficient as the traditional estimators are. Despite lack of gain in efficiency, there is no significant bias exhibited by the extreme-value estimators within the group. Another interesting aspect of the results in this group is that unlike the previous two groups, the average bias for the estimators is very high, though insignificant. The average bias for the traditional estimator is –20% to –22% for one-day estimation period, when annualized. Even for the extreme-value estimators, it ranges between –14% to –16%. At five-day estimation period, it ranges between –6 to –7% across the estimators and the stocks. Even at one-month estimation period, it ranges between –2.7% to –3.4% across the estimators and the stocks. Given these results, even though the extreme-value estimators compare favorably with their traditional counterparts, there is a need to empirically examine the distribution-characteristics of their returns and realized volatility to ascertain whether 5-minutes returns are appropriate as the realized volatility measure for them.

Insert Table 5 about here.

Our results in the study by and large, support the use of extreme-value estimators in Indian context. We do not find any significant bias in any of the extreme-value estimators when used to estimate volatility of individual stock, though the Parkinson estimator performs badly on this criterion for the index. The Parkinson estimator however, seems to perform well on the bias and the efficiency criteria for the individual stocks. Even though the gain in efficiency varies with the use of specific extreme-value estimator, all of them perform well compared to their traditional counterparts across estimation periods and assets. The average bias of the estimators across assets however, is fairly high (but, insignificant) over one-day estimation period and becomes less than 1%, when estimated over one-month. In case of illiquid stocks, the benefits of using the extreme-value estimators seem to be marginal, when compared with the traditional estimators.

5.0 Summary and Conclusions

Even though the volatility estimators using extreme-values (highest and lowest prices) observed during the trading on a given day have been proposed in the literature, their use by the practitioners and researchers has been rather limited. In this paper, we report the result of an empirical investigation in performance of some of these extreme-value estimators in the context of Indian Capital markets. Using high-frequency data set of 5-minute return series for a stock index (S&P CNX Nifty) and 10 constituent stocks to estimate the realized volatility, we compare both the traditional and the extreme-value estimators on a set of criteria defined to evaluate their bias and efficiency. We find that almost all the extreme-value estimators are free of bias when compared with the realized volatility and are less biased than the traditional volatility estimators. The only exception is the Parkinson estimator when used as an estimator for the index. The extreme-value estimators are also found to be more efficient than their traditional counterparts for the stocks as well as the index. The efficiency of the extreme-value estimators is however, less for relatively illiquid stocks and longer estimation period of one-month. Besides incorporating the usual performance criteria to evaluate bias and efficiency, we include a criterion for evaluating the predictive power of various estimators. The extreme-value estimators outperform their traditional counterparts on this criterion as well. Based on our results, we conclude that almost all the extreme-value estimators are bias-free and have higher efficiency than their traditional counterparts. They can be used safely for estimating the volatility of liquid assets. Though a specific estimator performs best for a particular asset, all of the proposed extreme-value estimators yield more efficient estimates than their traditional counterparts. Parkinson estimator however, can be avoided for longer (more than 5-days) estimation period for the index. Similarly, Rogers-Satchell estimator can be avoided for very short periods, if the underlying asset prices, based on summary prices related to opening, highest, lowest and closing prices, have been moving in one direction. In such a case, it may under-estimate the "true" volatility realized during the trading.

The volatility estimates have important applications in- (a) integrated volatility based risk management techniques such as VaR, (b) risk surveillance and monitoring by the regulators while setting margins or capital adequacy requirements, and (c) valuation/ pricing of OTC options or in option trading. Even asset allocation strategies by the investment managers may use volatility (to be precise, estimate of variance-covariance matrix of the assets) to assess the risks implied in their choices. While historically all these applications might have been relatively less important in Indian context, their use is increasing with the introduction of derivatives and increase in the sophistication-level of the participants and in the regulatory approach. This study focuses on the possible usefulness of the extreme value estimators for some of these applications. We acknowledge however, that the time-varying characteristic of volatility (GARCH effect) also needs to be studied for better understanding and for confidence in applications. Given the efficiency of the extreme-value estimators, it would be interesting to compare the ability of the two approaches in terms of forecast errors. Future research in this area could also compare their performance with "realized volatility" based models for forecasting volatility. As opposed to various GARCH and the realized volatility models, the extreme-value estimators are easy to compute and require much less data. In the applications for volatility estimate of a specific liquid asset, the use of extreme-value estimators could be particularly useful. Besides comparisons with the competing volatility models, our study can be extended by including more stocks particularly illiquid stocks and by evaluating the performance of Garman-Klass and Rogers-Satchell estimators modified for discrete-trading (given in equations 8 and 9). This would require using information on number of trades. Another interesting area of empirical work would be to investigate the distribution characteristics of the realized volatility in Indian markets, as it may shed some light on the negative average bias encountered across the assets in this study.

Table 1: The Stocks used in the Study and Their Volatility and Liquidity Characteristics

Stock	Average Volume (# of shares / day)	Annualize Daily Realized Volatility	Remarks
Hindustan Lever Ltd.	804881	35.389%	Liquid, Less Volatile
Reliance Industries Ltd.	6522606	38.672%	Liquid, Less Volatile
Satyam Computers Ltd.	7307391	63.276%	Liquid, Highly Volatile
Larsen & Toubro Ltd.	2388425	49.284%	Liquid, Moderately Volatile
Infosys Technologies Ltd.	393371	50.277%	Liquid, Moderately Volatile
Novartis Ltd.	11823	58.644%	Illiquid, Moderately Volatile
Indian Hotels Ltd.	19876	55.347%	Illiquid, Moderately Volatile
NIIT Ltd.	637410	65.459%	Liquid, Highly Volatile
Zee Telefilms Ltd.	6090560	70.482%	Liquid, Highly Volatile
S Kline Beech. Cons. Ltd.	14642	50.734%	Illiquid, Moderately Volatile

Table 2: Descriptive Statistics for the Annualized Daily Realized Volatility

(Number of Observations- 737)

Index	**Mean**	**Std. Deviation**	**Skewness**	**Kurtosis**
S&P CNX Nifty	26.0158%	14.9464%	2.467	9.449
Stocks				
Hindustan Lever Ltd.	35.3894%	19.7026%	2.097	6.519
Reliance Industries Ltd.	38.6722%	20.0681%	2.007	6.736
Satyam Computers Ltd.	63.2764%	30.6497%	2.099	8.028
Larsen & Toubro Ltd.	49.2841%	21.6086%	2.009	9.840
Infosys Technologies Ltd.	50.2770%	29.3351%	1.831	5.892
Novartis Ltd.	58.6472%	29.6923%	1.928	6.832
Indian Hotels Ltd.	55.3466%	24.6863%	1.252	2.371
NIIT Ltd.	65.4591%	35.7141%	1.780	5.326
Zee Telefilms Ltd.	70.4816%	36.2439%	1.746	4.996
S Kline Beech. Cons. Ltd.	50.7336%	26.0278%	1.347	2.458

Table 3: Performance of Volatility Estimators for the Index (S&P CNX Nifty)

Panel A: Volatility Estimates[?] over One-day Period

(Number of Observations- 737)

Estimator	Bias	Variance	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.003444	0.000144	-0.209291	0.000108	0.007938	0.000139
std.error	0.009805		0.545990	0.000196	0.006702	0.000346
Parkinson	-0.006073	**0.000036**	-0.356176	0.000065	0.006191	0.000098
std.error	0.005254		0.178974	0.000146	0.005114	0.000272
Garman-Klass	-0.001534	0.000071	-0.067693	**0.000029**	**0.003619**	**0.000081**
std.error	0.005138		0.251473	0.000094	0.003955	0.000246
Rogers-Satchell	**-0.001521**	0.000082	**-0.067535**	0.000040	0.004060	0.000101
std.error	0.006153		0.290640	0.000153	0.004865	0.000306

Panel B: Volatility Estimates[?] over Five-day Period

(Number of Observations- 147)

Estimator	Bias	Variance	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.001649	0.000071	-0.099506	0.000022	0.003478	0.000076
std.error	0.004359		0.242577	0.000038	0.003093	0.000192
Traditional Adj.	-0.001975	0.000076	-0.120151	0.000032	0.004145	0.000090
std.error	0.005292		0.278473	0.000073	0.003825	0.000234
Parkinson	-0.006296	**0.000024**	-0.358154	0.000053	0.006296	0.000083
std.error	0.003659		0.086853	0.000072	0.003659	0.000167
Garman-Klass	-0.001535	0.000047	-0.073482	0.000011	0.002227	**0.000049**
std.error	0.002889		0.130167	0.000026	0.002392	0.000119
Rogers-Satchell	**-0.001265**	0.000050	**-0.057431**	0.000012	0.002325	**0.000049**
std.error	0.003249		0.145748	0.000043	0.002593	0.000118
Yang-Zhiang	-0.001350	0.000049	-0.064940	**0.000010**	**0.002092**	**0.000049**
std.error	0.002833		0.129719	0.000029	0.002335	0.000118

[?] The volatility estimates reported here have not been annualized. For converting them in % annualized volatility, the reported volatility need to be multiplied with $(N)^{1/2} * 100$. N is 250 for one-day period, 50 for 5-days period and 12 for one-month period. The same factors will also scale up the reported Bias and Mean absolute difference while Relative Bias will remain unaffected. The Mean Square Error needs to be scaled up by multiplying with N instead of its square root.

Panel C: Volatility Estimates[?] over Calendar Month

(Number of Observations- 36)

Estimator	Bias	Variance	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.001244	0.000040	-0.063889	0.000010	0.002312	0.000059
std.error	0.002886		0.137101	0.000016	0.002103	0.000091
Traditional Adj.	**-0.001118**	0.000041	**-0.057564**	0.000009	0.002221	0.000059
std.error	0.002862		0.136858	0.000016	0.002098	0.000090
Parkinson	-0.006536	**0.000016**	-0.359917	0.000051	0.006536	0.000078
std.error	0.002919		0.047094	0.000052	0.002919	0.000118
Garman-Klass	-0.001664	0.000033	-0.084305	0.000006	0.001798	0.000040
std.error	0.001692		0.070527	0.000010	0.001544	0.000069
Rogers-Satchell	-0.001391	0.000036	-0.069543	0.000006	0.001692	**0.000040**
std.error	0.001945		0.086247	0.000012	0.001683	0.000069
Yang-Zhiang	-0.001353	0.000035	-0.067882	**0.000004**	**0.001537**	0.000041
std.error	0.001626		0.071685	0.000009	0.001449	0.000070

The numbers underlined are the least among all the estimators on that criterion.

Table 4: Performance of Volatility Estimators for the Index (S&P CNX Nifty) in Each of the Three Years of the Study

Panel A: Volatility Estimates[?] over One-day Period

Estimator	Year	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	1999	-0.002148	-0.163187	0.000083	0.007278	0.000130
	2000	-0.005179	-0.240401	0.000160	0.009681	0.000173
	2001	-0.003044	-0.226004	0.000082	0.006862	0.000114
Parkinson	1999	-0.005088	-0.340501	0.000042	0.005202	0.000073
	2000	-0.007675	-0.373562	0.000093	0.007724	0.000130
	2001	-0.005480	-0.354965	0.000059	0.005673	0.000092
Garman-Klass	1999	-0.001099	-0.057232	0.000023	0.003175	0.000062
	2000	-0.002112	-0.090388	0.000038	0.004362	0.000104
	2001	-0.001404	-0.055653	0.000025	0.003330	0.000080
Rogers-Satchell	1999	-0.001330	-0.065779	0.000036	0.003663	0.000072
	2000	-0.001886	-0.085878	0.000054	0.004997	0.000137
	2001	-0.001350	-0.050800	0.000030	0.003525	0.000096

[?] The volatility estimates reported here have not been annualized.

Panel B: Volatility Estimates[?] over Five-day Period

Estimator	Year	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	1999	<u>-0.000145</u>	-0.024803	0.000016	0.002980	0.000081
	2000	-0.003240	-0.155630	0.000033	0.004601	0.000067
	2001	-0.003222	-0.160843	0.000032	0.004554	0.000067
Adj. Traditional	1999	-0.000156	<u>-0.024303</u>	0.000022	0.003154	0.000093
	2000	-0.003715	-0.177804	0.000050	0.005464	0.000086
	2001	-0.003781	-0.190312	0.000049	0.005500	0.000086
Parkinson	1999	-0.005113	-0.328868	0.000032	0.005113	0.000065
	2000	-0.008047	-0.386850	0.000079	0.008047	0.000104
	2001	-0.007835	-0.384329	0.000075	0.007835	0.000100
Garman-Klass	1999	-0.000894	-0.044436	<u>0.000008</u>	<u>0.001737</u>	0.000042
	2000	-0.002241	-0.108176	0.000013	0.002687	<u>0.000050</u>
	2001	-0.002091	-0.101786	0.000012	0.002571	<u>0.000048</u>
Rogers-Satchell	1999	-0.000967	-0.041550	0.000014	0.002003	<u>0.000040</u>
	2000	<u>-0.001607</u>	<u>-0.081460</u>	0.000011	0.002706	0.000051
	2001	<u>-0.001476</u>	<u>-0.074792</u>	0.000011	0.002578	0.000049
Yang-Zhiang	1999	-0.000870	-0.039485	0.000009	0.001749	0.000040
	2000	-0.001860	-0.092994	<u>0.000010</u>	<u>0.002373</u>	0.000050
	2001	-0.001752	-0.088621	<u>0.000009</u>	<u>0.002266</u>	0.000048

Table 5: Performance of Volatility Estimators[?] for the Stocks

Panel A- Liquid and Relatively Less Volatile Stocks

Estimation Period: One-day

Hindustan Lever (Number of observations-746)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.004590	-0.219088	0.000223	0.011210	0.000309
Parkinson	-0.000828	-0.034944	**0.000046**	**0.004742**	**0.000160**
Garman-Klass	**-0.000664**	**-0.021128**	0.000055	0.004749	0.000173
Rogers-Satchell	-0.000831	-0.026098	0.000089	0.005630	0.000210

Reliance (Number of observations-746)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.003546	-0.180627	0.000279	0.012908	0.000320
Parkinson	**-0.000459**	**-0.024079**	**0.000051**	0.005055	**0.000150**
Garman-Klass	-0.000778	-0.023375	0.000059	**0.004911**	0.000176
Rogers-Satchell	-0.001409	-0.038576	0.000114	0.006343	0.000250

Larsen & Toubro (Number of observations-746)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.005899	-0.212646	0.000415	0.016173	0.000528
Parkinson	-0.000977	-0.025256	**0.000073**	0.006226	**0.000205**
Garman-Klass	**-0.000956**	**-0.011638**	0.000086	**0.006166**	0.000213
Rogers-Satchell	-0.001630	-0.024569	0.000156	0.007649	0.000281

Infosys Technologies (Number of observations-746)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.003272	**-0.128096**	0.0005315	0.0164795	0.000724
Parkinson	**-0.000778**	0.228249	**0.0001055**	0.0070742	**0.000401**
Garman-Klass	-0.001849	0.255509	0.0001178	**0.0070137**	0.000426
Rogers-Satchell	-0.002778	0.352555	0.0002284	0.0090093	0.000545

[?] The volatility estimates reported here have not been annualized. For converting them in % annualized volatility, the reported volatility need to be multiplied with $(N)^{1/2} * 100$. N is 250 for one-day period, 50 for 5-days period and 12 for one-month period. The same factors will also scale up the reported Bias and Mean absolute difference while Relative Bias will remain unaffected. The Mean Square Error needs to be scaled up by multiplying with N instead of its square root.

Estimation Period: Five-day

Hindustan Lever (Number of observations-149)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.001668	-0.071267	0.000055	0.005345	0.000114
Adj. Traditional	-0.002423	-0.104164	0.000069	0.005953	0.000135
Parkinson	-0.000444	-0.011953	**0.000014**	**0.002708**	**0.000085**
Garman-Klass	-0.000359	**-0.004160**	0.000022	0.003205	0.000094
Rogers-Satchell	**-0.000212**	0.005995	0.000037	0.003808	0.000110

Reliance (Number of observations-149)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	**0.000005**	-0.019656	0.000069	0.006473	0.000153
Adj. Traditional	-0.000338	-0.025423	0.000086	0.006945	0.000185
Parkinson	0.000257	0.011335	**0.000014**	**0.002747**	**0.000099**
Garman-Klass	-0.000112	**0.004906**	0.000019	0.002968	0.000105
Rogers-Satchell	-0.000142	0.010029	0.000033	0.003833	0.000117

Larsen & Toubro (Number of observations-149)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.001380	-0.053980	0.000099	0.007992	0.000225
Adj. Traditional	-0.002196	-0.076633	0.000120	0.008978	0.000265
Parkinson	**-0.000431**	-0.006162	**0.000022**	**0.003599**	0.000118
Garman-Klass	-0.000560	-0.002849	0.000029	0.003648	**0.000111**
Rogers-Satchell	-0.000666	**-0.002522**	0.000041	0.004303	0.000116

Infosys Technologies (Number of observations-149)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	0.001611	0.050516	0.000130	0.008737	0.000260
Adj. Traditional	0.001367	0.050783	0.000154	0.009222	0.000294
Parkinson	**-0.000252**	**0.001871**	**0.000028**	**0.004156**	**0.000158**
Garman-Klass	-0.001625	-0.037083	0.000037	0.004487	0.000166
Rogers-Satchell	-0.001772	-0.041328	0.000058	0.005417	0.000187

Estimation Period: One-month

Hindustan Lever (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.000971	-0.041976	0.000014	0.002655	**0.000052**
Adj. Traditional	-0.000992	-0.044163	0.000014	0.002712	0.000052
Parkinson	-0.000378	-0.010932	**0.000008**	**0.002052**	0.000054
Garman-Klass	-0.000297	**-0.004912**	0.000014	0.002530	0.000061
Rogers-Satchell	**-0.000032**	0.010044	0.000023	0.003077	0.000075

Reliance (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	0.001056	0.030310	0.000025	0.003909	0.000149
Adj. Traditional	0.001364	0.041660	0.000028	0.004029	0.000158
Parkinson	0.000373	0.016869	**0.000006**	**0.001823**	0.000100
Garman-Klass	-0.000068	**0.004982**	0.000009	0.002105	0.000090
Rogers-Satchell	**-0.000003**	0.012302	0.000013	0.002652	**0.000088**

Larsen & Toubro (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	**-0.000303**	-0.010639	0.000038	0.004399	0.000118
Adj. Traditional	-0.000516	-0.013934	0.000044	0.004801	0.000123
Parkinson	-0.000491	**-0.010349**	**0.000009**	**0.001955**	0.000075
Garman-Klass	-0.000754	-0.015497	0.000012	0.002054	**0.000071**
Rogers-Satchell	-0.000859	-0.017340	0.000016	0.002570	0.000072

Infosys Technologies (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	0.002965	0.099812	0.000044	0.005520	0.000194
Adj. Traditional	0.003181	0.105520	0.000048	0.005707	0.000204
Parkinson	**-0.000239**	**-0.000318**	**0.000008**	**0.002231**	**0.000145**
Garman-Klass	-0.001768	-0.048274	0.000014	0.002979	0.000147
Rogers-Satchell	-0.001812	-0.052971	0.000021	0.003605	0.000158

Panel B- Liquid and Relatively More Volatile Stocks

Estimation Period: One-day

Satyam Computers (Number of observations-746)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.003259	-0.092651	0.000682	0.019904	0.000796
Parkinson	**-0.000513**	**-0.006354**	**0.000123**	**0.008125**	**0.000353**
Garman-Klass	-0.001827	-0.031089	0.000149	0.008173	0.000404
Rogers-Satchell	-0.003284	-0.061837	0.000290	0.010829	0.000558

NIIT (Number of observations-746)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.007218	-0.189523	0.000783	0.020871	0.000870
Parkinson	**-0.001910**	-0.038093	**0.000145**	**0.008673**	**0.000425**
Garman-Klass	-0.002301	**-0.037282**	0.000168	0.008969	0.000495
Rogers-Satchell	-0.003170	-0.051327	0.000337	0.011261	0.000700

Zee Telefilms (Number of observations-745)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	-0.006446	-0.167164	0.000857	0.022511	0.001094
Parkinson	**-0.002221**	**0.060889**	**0.000164**	**0.009367**	**0.000499**
Garman-Klass	-0.003365	0.068311	0.000213	0.009536	0.000567
Rogers-Satchell	-0.004735	0.095094	0.000425	0.012565	0.000792

Estimation Period: Five-day

Satyam Computers (Number of observations-149)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	0.002364	0.060772	0.000145	0.009189	0.000309
Adj. Traditional	0.001982	0.051344	0.000172	0.010219	0.000377
Parkinson	**0.000230**	**0.010556**	**0.000031**	**0.003872**	**0.000201**
Garman-Klass	-0.001186	-0.022658	0.000043	0.004359	0.000213
Rogers-Satchell	-0.001590	-0.032866	0.000067	0.005565	0.000234

NIIT (Number of observations-149)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.001986	-0.058720	0.000187	0.010378	0.000395
Adj. Traditional	-0.002955	-0.086014	0.000278	0.012727	0.000533
Parkinson	**-0.001268**	-0.025191	**0.000038**	**0.004680**	0.000224
Garman-Klass	-0.001643	-0.027280	0.000048	0.005051	**0.000222**
Rogers-Satchell	-0.001328	**-0.017204**	0.000072	0.006253	0.000236

Zee Telefilms (Number of observations-149)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	**-0.000271**	**-0.012339**	0.000184	0.010189	0.000382
Adj. Traditional	-0.000774	-0.025654	0.000241	0.011901	0.000475
Parkinson	-0.001420	-0.019312	**0.000052**	**0.005175**	**0.000248**
Garman-Klass	-0.002591	-0.037644	0.000079	0.005849	0.000276
Rogers-Satchell	-0.002627	-0.035406	0.000120	0.007388	0.000315

Estimation Period: One-month

Satyam Computers (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	0.003464	0.091407	0.000055	0.005660	0.000235
Adj. Traditional	0.003710	0.097718	0.000062	0.005999	0.000246
Parkinson	**0.000366**	0.017042	**0.000010**	**0.001972**	0.000149
Garman-Klass	-0.001054	**-0.016498**	0.000015	0.002448	0.000133
Rogers-Satchell	-0.001318	-0.023787	0.000023	0.003029	**0.000132**

NIIT (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	**-0.000794**	-0.030918	0.000069	0.006002	0.000480
Adj. Traditional	-0.001083	-0.035817	0.000069	0.006099	0.000475
Parkinson	-0.001382	-0.034342	**0.000011**	**0.002477**	0.000297
Garman-Klass	-0.001815	-0.039607	0.000013	0.002746	0.000246
Rogers-Satchell	-0.001336	**-0.027112**	0.000017	0.002987	**0.000227**

Zee Telefilms (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	**0.001285**	0.033271	0.000051	0.005606	0.000195
Adj. Traditional	0.001548	0.038202	0.000053	0.005683	0.000208
Parkinson	-0.001496	**-0.022616**	**0.000020**	**0.003065**	**0.000173**
Garman-Klass	-0.002803	-0.049301	0.000030	0.004106	0.000189
Rogers-Satchell	-0.002693	-0.048090	0.000039	0.005101	0.000214

Panel C- Relatively Illiquid Stocks

Estimation Period: One-day

Indian Hotels (Number of observations-746)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.013710	-0.396346	0.000513	0.018211	0.000693
Parkinson	**-0.009580**	**-0.267010**	**0.000186**	**0.011004**	**0.000376**
Garman-Klass	-0.009720	-0.267130	0.000207	0.011456	0.000388
Rogers-Satchell	-0.009999	-0.271203	0.000281	0.012606	0.000455

SmithKline Beecham (Number of observations-746)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.012358	-0.402294	0.000463	0.016928	0.000642
Parkinson	**-0.009127**	-0.283579	**0.000181**	**0.010677**	**0.000365**
Garman-Klass	-0.009418	-0.284987	0.000200	0.011077	0.000372
Rogers-Satchell	-0.009452	**-0.279729**	0.000263	0.011929	0.000425

Novartis (Number of observations-462)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.012528	-0.340431	0.000588	0.018744	0.000857
Parkinson	**-0.009048**	**-0.236926**	**0.000226**	**0.011403**	**0.000511**
Garman-Klass	-0.009582	-0.248524	0.000257	0.011873	0.000531
Rogers-Satchell	-0.010251	-0.263982	0.000367	0.013453	0.000630

Estimation Period: Five-day

Indian Hotels (Number of observations-149)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.010460	-0.287410	0.000216	0.012037	0.000316
Adj. Traditional	-0.011619	-0.322713	0.000258	0.013183	0.000374
Parkinson	-0.009281	-0.251954	**0.000118**	0.009487	0.000213
Garman-Klass	-0.009324	-0.251878	0.000120	0.009518	**0.000212**
Rogers-Satchell	**-0.008826**	**-0.237829**	0.000118	**0.009177**	0.000215

SmithKline Beecham (Number of observations-149)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.009056	-0.269997	0.000200	0.011478	0.000283
Adj. Traditional	-0.008912	-0.274911	0.000216	0.011984	0.000314
Parkinson	-0.008739	-0.256867	**0.000116**	**0.009082**	**0.000214**
Garman-Klass	-0.009174	-0.268066	0.000127	0.009610	0.000227
Rogers-Satchell	**-0.008576**	**-0.248837**	0.000129	0.009351	0.000233

Novartis (Number of observations-92)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.009128	-0.233213	0.000214	0.011074	0.000348
Adj. Traditional	-0.008903	-0.229771	0.000233	0.011937	0.000379
Parkinson	-0.008866	-0.220185	**0.000135**	0.009306	**0.000259**
Garman-Klass	-0.009209	-0.225750	0.000146	0.009491	0.000263
Rogers-Satchell	**-0.008709**	**-0.210732**	0.000149	**0.009258**	0.000269

Estimation Period: One-month

Indian Hotels (Number of observations-36)

Estimator	**Bias**	**Relative Bias**	**Mean Square Error**	**Mean Absolute Difference**	**MSE of one-period ahead forecast**
Traditional	-0.009324	-0.247776	0.000134	0.009569	0.000193
Adj. Traditional	-0.009911	-0.262958	0.000144	0.010145	0.000206
Parkinson	-0.009163	-0.242770	0.000101	0.009163	0.000166
Garman-Klass	-0.009263	-0.245216	0.000102	0.009263	0.000169
Rogers-Satchell	**-0.008621**	**-0.227102**	**0.000090**	**0.008621**	**0.000159**

SmithKline Beecham (Number of observations-36)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	**-0.007834**	**-0.215427**	0.000110	**0.008285**	0.000180
Adj. Traditional	-0.007952	-0.220083	0.000115	0.008411	0.000189
Parkinson	-0.008508	-0.236555	0.000093	0.008508	0.000160
Garman-Klass	-0.008934	-0.249956	0.000099	0.009029	0.000165
Rogers-Satchell	-0.008154	-0.227197	**0.000086**	0.008419	**0.000155**

Novartis (Number of observations-22)

Estimator	Bias	Relative Bias	Mean Square Error	Mean Absolute Difference	MSE of one-period ahead forecast
Traditional	**-0.008058**	**-0.193497**	**0.000101**	**0.008524**	**0.000178**
Adj. Traditional	-0.008543	-0.205981	0.000105	0.008626	0.000196
Parkinson	-0.008960	-0.216677	0.000103	0.008960	0.000208
Garman-Klass	-0.009415	-0.228126	0.000114	0.009415	0.000230
Rogers-Satchell	-0.008870	-0.213720	0.000110	0.008870	0.000232

References:

1. Andersen, T.G. and T. Bollerslev, 1998, "Answering the Skeptics: Yes, Standard Volatility Models Do Provide Accurate Forecasts", *International Economic Review 39,* 885-905.

2. Andersen, T.G., T. Bollerslev, F.X. Diebold and P. Labys, 1999, "Microstructure Bias and Volatility Signatures", Unpublished Manuscript, New York University.

3. Andersen, T.G., T. Bollerslev, F.X. Diebold and P. Labys, 2001a, "The distribution of Realized Exchange Rate Volatility", *Journal of American Statistical Association 96,* 42-55.

4. Andersen, T.G., T. Bollerslev, F.X. Diebold and H. Ebens, 2001b, "The distribution of Stock Return Volatility", *Journal of Financial Economics 61(1),* 43-76.

5. Beckers, S., 1983, "Variances of Security Price Returns based on High, Low and Closing Prices", *Journal of Business 56,* 97-112.

6. Cho, D.C. and E.W. Frees, 1988, "Estimating the Volatility of Discrete Stock Prices", *Journal of Finance 43,* 451-466.

7. Engle, R.F., 1982, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation", *Econometrica 50,* 987-1008.

8. Feller, W., 1951, "The Asymptotic Distribution of the Range of sums of Independent Random Variables", *Annals of Mathematical Statistics 22,* 427-432.

9. Garman, M.B. and M.J. Klass, 1980, "On the estimation of Security Price Volatilities from Historical data", *Journal of Business 53,* 67-78.

10. Kunitomo, N., 1992, "Improving Parkinson Method of estimating Security Price Volatilities", *Journal of Business 65,* 295-302.

11. Li, K. and D. Weinbaum, 2000, "The Empirical Performance of Alternative Extreme Value Volatility Estimators", *Working Paper,* Stern School of Business, New York (at www.stren.nyu.edu)

12. Marsh, T. and E. Rosenfeld, 1986, "Non-Trading, Market Making, and Estimates of Stock Price Volatility", *Journal of Financial Economics 15,* 359-372.

13. Parkinson, M., 1980, "The extreme value method for estimating the variance of the rate of return", *Journal of Business 53,* 61-65.

14. Rogers, L.C.G. and S.E. Satchell, 1991, "Estimating Variance from High, Low and Closing Prices", *Annals of Applied Probability 1,* 504-512.

15. Spurgin, R.B. and T. Schneeweis, 1999, "Efficient Estimation of Intra-day Volatility: A Method-of-Moments approach incorporating Trading Range", in *Financial Markets Tick by Tick,* Ed: P. Lequeux, John Wiley & Sons, England.

16. Wiggins, J.B., 1991, "Empirical Tests of the Bias and Efficiency of the Extreme-Value Variance Estimator for Common Stocks", *Journal of Business 64,* 417-432.

17. Wiggins, J.B., 1992, "Estimating the Volatility of S&P Futures Prices Using the Extreme-Value Method", *Journal of Futures Markets 12,* 265-273.

18. Yang, D. and Q. Zhang, 2000, "Drift Independent Volatility Estimation based on High, Low, Open and Close Prices", *Journal of Business 73,* 477-492.